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Overfitting and forecasting: linear versus non-linear time series models

by

Yamei Liu

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Major Professor: Walter Enders

Iowa State University

Ames, Iowa

2000

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This is certify that the Doctoral dissertation of

Yamei Liu

has met the dissertation requirements of Iowa State University

Signatures have been redacted for privacy.

To

My husband, Shaobin,

my son, Rodman

and my parents

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CHAPTER 1 INTRODUCTION

In the most recent two decades or so, there has been great interest in the examination of non-linear time series models. The interest mainly results from a large literature documenting the asymmetric behavior of many macroeconomic variables. Neftci (1984) presented his seminal paper by showing that U.S. unemployment displays asymmetric adjustment over the course of the business cycle. Falk (1986) applied Neftci's method to real GNP, investment and productivity in U.S. and industrial production in Canada, France, Italy, Germany and the United Kingdom and found little evidence of asymmetry. Non-linearities have been found in Divisia monetary aggregates (Barnett and Chen, 1988). Brock and Sayers (1988) and Scheinkman and LeBaron (1989) found little evidence of non-linear dynamics in output, but important non-linearities in industrial production and employment series. Ashley and Patterson (1989) found non-linearities in industrial production and employment series, too. Asymmetric processes of U.S. sales, production and inventories adjusting toward their long-run equilibrium relationships were shown by Granger and Lee (1989). Beaudry and Koop (1993) found that negative innovations to GNP are much less persistent than positive ones. The positive correlation of asymmetric effects of monetary shocks on output and expected inflation was reported by Rhee and Rich (1995). Tinsley and Krieger (1997) found that the negative deviations from the trend of production are larger than the positive ones and that price levels are more ready to increase than to decrease.

Recently, much work has been done to specify the type of non-linearity, model the non-linearity and explain the reasons of non-linearity. Teräsvirta and Anderson (1992) modeled industrial production in 13 OECD countries as smooth transition autoregressive models and found that industrial production responds more sharply to negative shocks than to positive shocks. Sichel (1993) used "deepness" and "sharpness" to describe the asymmetric behavior over the business cycle. Deepness means that troughs are more pronounced than peaks and sharpness means that contractions are steeper than expansions. Then the evidence of deepness and sharpness in U.S. unemployment, only deepness in U.S. industrial production and GNP was found. Potter (1995) modeled changes in real U.S. GNP as a threshold adjustment process and found that the post-1945 U.S. economy is significantly more stable than the pre-1945 U.S. economy. Shen and Hakes (1995) applied a threshold autoregressive model to the reaction function of the central bank of Taiwan and found that the central bank responds asymmetrically to its policy objectives when the severity of inflation differs. Ramsey and Rothman (1996) found both steepness and deepness in several of the Nelson and Plosser (1982) data series. Ball and Mankiw (1994) presented a menu-cost model with positive trend inflation to show that prices respond more strongly to positive shocks than to negative shocks.

However, little has been studied about in-sample estimating and out-of-sample forecasting performance of a set of non-linear time series models for those time series documented to have asymmetric behavior. Rothman (1998) found that several non-linear forecasts do dominate the linear forecast of the unemployment rates and that the results are sensitive to whether a stationarity-inducing transformation is applied to the potentially nonstationary unemployment rate series. Thus, one purpose of this study is to see how the non-linearities can be estimated and forecasted by different non-linear time series models. Since an extensive literature has examined the asymmetric adjustments of the term structure of interest rates and the spread of wholesale and farm pork prices in U.S., I will work on these series.

A related question is whether the series is overfitted. When we estimate and forecast a time series by a set of linear and non-linear time series models and find that non-linear time series models dominate linear models, can we say that non-linear models work better for the non-linear time series than linear models? Is it possible that the data is overfitted? In practice, when we fit data, we search for the "best" model from a set of some predetermined models by some criteria, for example, goodness-of-fit, Akaike information criterion (AIC), Schwartz Bayesian criterion (SBC), etc. Whether the data is overfitted is never answered. Some think that overfitted models should predict out-of-sample poorly. Is this the case? Can we use mean squared prediction error (MSPE) as a criterion to choose correct models? I am going to use a Monte Carlo simulation to study overfitting and forecasting in the univariate time series context.

This dissertation is organized as follows. Chapter 2 reviews various time series models used in this dissertation. Chapter 3 studies overfitting and forecasting by a Monte Carlo simulation. Chapter 4 applies different time series models to the term structure of interest rates in U.S. to study their in-sample estimating and out-of-sample forecasting performances. Chapter 5 applies different time series models to the spread of wholesale and farm pork prices in U.S. to study their in-sample estimating and out-of-sample forecasting performances. Conclusions of this study and suggestions for future research are provided in Chapter 6 .

CHAPTER 2 VARIOUS TIME SERIES MODELS

Autoregressive integrated moving average (ARIMA) models, threshold autoregressive (TAR) models, exponential autoregressive (EAR) models, smoothing transition autoregressive (STAR) models, bilinear autoregressive (BL) models, generalized autoregressive (GAR) models and generalized autoregressive conditional heteroskedasticity (GARCH) models are briefly reviewed in this chapter. All these models are parametric univariate time series. All the non-linear time series models are state-dependent in the sense that their dynamics vary with their past processes.

2.1 Autoregressive Integrated Moving Average (ARIMA) Models

Let y_t be a time series. The objective is to model the conditional mean of y_t parametrically given previous observations. Let

$$y_t = f\,(y_{t-i}, e_{t-j}, i = 1, \ldots, p, j = 1, \ldots, q;\ \theta) + e_t \qquad (2.1)$$

where $\{e_t, t \geq 1\}$ is a series of white noise process with zero mean. Then, the conditional mean of y_t is given by

$$E(\, y_t \mid y_{t-i}, e_{t-j}, i = 1, \ldots, p, j = 1, \ldots, q) = f\,(y_{t-i}, e_{t-j}, i = 1, \ldots, p, j = 1, \ldots, q;\ \theta) \qquad (2.2)$$

where the function f(.) is known and θ is an unknown parameter vector to be estimated.

Until recently, most of time series modeling has been confined to linear models, i.e., autoregressive integrated moving average (ARIMA) models. The standard ARIMA (p, d, q) model has the following form:

$$\nabla^d y_t = \sum_{i=1}^{p} \theta_{1i} \nabla^d y_{t-i} + e_t + \sum_{i=1}^{q} \theta_{2i} e_{t-i} \tag{2.3}$$

where $\nabla^d y_t$ means that y_t has been differenced d times; thus, y_t has d unit roots and is said to be integrated of order d, denoted I(d) (see Engle and Granger, 1987). In practice, d is usually 0, 1 or 2. The ARIMA (p, d, q) models have been extensively analyzed and popularized by Box and Jenkins (1970), where model specification, estimation and diagnostic checking were analyzed. The model specification phase consists in determining d, the degree of differencing needed to make y_t stationary, and in determining p and q. The estimation phase yields estimates of the parameters $\{\theta_{1i}; i = 1, ..., p\}$, $\{\theta_{2i}; i = 1, ..., q\}$ and $\sigma^2 = E(e_i^2)$ for given values of p and q. The model verification is used to check whether the estimated residual process $\{\hat{e}_t\}$ can be identified as a white noise process. Since these models are very popular, I am not going to discuss the specification, estimation and verification in detail here. For simplicity, when I refer to y_t later on, I assume it is I(0), possibly after some prior differencing or other transformation. Thus, I use autoregressive moving average (ARMA) models rather than ARIMA models.

2.2 Threshold Autoregressive (TAR) Models

These are sometimes also called self-exciting threshold autoregressive (SETAR) models, developed by Tong (1983). Tong (1990) defined TAR (k; p,...,p), where p is repeated k times, as:

$$y_t = \theta_0^{(j)} + \sum_{i=1}^{p} \theta_i^{(j)} y_{t-i} + e_t^{(j)} \quad \text{if } y_{t-d} \in R_j;\ j = 1, \ldots, k \tag{2.4}$$

where $R_j = (r_{j-1}, r_j]$, $r_0 = -\infty$, $r_k = +\infty$, $r_0 < r_1 < \ldots < r_k$, $\{r_i, i = 1, \ldots, k-1\}$ are real numbers and called thresholds, y_{t-d} is the transition variable, and d is called the delay variable. Thus, the real line R is partitioned as: $R = R_1 \cup R_2 \ldots \cup R_k$ with an AR model in each regime. The parameter values change abruptly through time due to a switching rule, which depends on an earlier value of the series. The major features of the models are asymmetric limit cycles, amplitude dependent frequencies and jump resonances[1].

A much simpler form of TAR models can be defined as:

$$y_t = \theta_0 + \rho_1 y_{t-1} I_t + \rho_2 y_{t-1}(1 - I_t) + \sum_{i=2}^{p} \theta_i y_{t-i} + e_t \tag{2.5}$$

where I_t is the Heaviside indicator function such that

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} \geq 0 \\ 0 & \text{if } y_{t-1} < 0 \end{cases} \tag{2.6}$$

These models can capture the deepness phenomenon if autoregressive process is more persistent for the negative phase than for the positive phase, i.e., $|\rho_1| < |\rho_2|$.

The above (2.5) models become momentum threshold autoregressive (M-TAR) models (Enders and Granger, 1998) if the Heaviside indicator function I_t is defined as:

$$I_t = \begin{cases} 1 & if\ \Delta y_{t-1} \geq 0 \\ 0 & if\ \Delta y_{t-1} < 0 \end{cases} \tag{2.7}$$

The M-TAR models can capture the sharpness phenomenon if the autoregressive decay is faster for the negative change of a variable than for the positive change of a variable, i.e., $|\rho_1| < |\rho_2|$.

We can easily extend (2.5) to the more general form of (2.8):

$$y_t = \theta_{10} I_t + \theta_{20}(1 - I_t) + \sum_{i=1}^{p} \theta_{1i} y_{t-i} I_t + \sum_{i=1}^{p} \theta_{2i} y_{t-i}(1 - I_t) + e_t \tag{2.8}$$

In (2.8), the residual is restricted to be the same across regimes, which is different from (2.4). (2.8) is useful if we want to restrict certain estimated coefficients to be equal across regimes. Also, (2.6) and (2.7) can be extended to the Heaviside indicator functions with more than two regimes. I use (2.8), rather than (2.4), to estimate TAR models in this dissertation.

Chan (1993) showed that the conditional least squares estimator of a stationary ergodic[2] TAR model is strongly consistent. Thus grid search can be used to find the threshold(s) and then get the conditional least squares estimator. The procedure is described as follows:

I. Decide the range of the order and delay variable.

II. For a fixed threshold and delay variable, use conditional least squares to estimate the TAR model.

III. The above steps repeated for each delay variable and each threshold. The threshold and delay variable are chosen to minimize the conditional sum of square errors. Usually, the threshold is chosen from a range of the observations, for example, from the 10-th percentile to the 90-th percentile of the series.

Also, the arranged autoregression method was proposed by Tsay (1989) to test and build TAR models. The procedure is described as follows.

I. Select the order p and the possible set S for d. The partial autocorrelation function (PACF), Akaike information criterion (AIC) and Schwartz Bayasian criterion (SBC)[3] can be used to choose p; d is usually assumed to be less than or equal to p.

II. Order the observations, fit the arranged autoregressions for the given p and every element d in S, and perform a threshold non-linearity F-test. If linearity is rejected for more than one d, select d by minimizing the p-value of the F-test.

III. For given values of p and d, locate threshold values by using the scatter plots of the standardized predictive residuals versus y_{t-d}, or/and predictive residuals versus y_{t-d} or/and t-ratios of recursive estimates of a significant AR coefficient versus y_{t-d}.

IV. If necessary, refine the order p and threshold values in each regime using linear autoregressive techniques.

This class of models has been used in many studies. Tong and Lim (1980) showed that TAR models can capture the asymmetric behavior exhibited in the annual Wolf's sunspot data and Canadian lynx data. Shen and Hakes (1995) estimated a TAR model for the reaction function of the central bank of Taiwan. Potter (1995) modeled real U.S. GNP as a two-regime TAR model, and Enders and Granger (1998) used TAR and M-TAR models to study the asymmetric adjustments of the term structure of interest rates in U.S.. Tsay (1989) applied TAR models to the sunspot and Canadian lynx series. For example, he estimated the logged Canadian lynx data[4] as:

$$\begin{aligned} y_t &= 0.083 + 1.096 y_{t-1} + e_t^{(1)}, \ \textit{if}\ y_{t-2} \le 2.373 \\ &= 0.63 + 0.96 y_{t-1} - 0.11 y_{t-2} + 0.23 y_{t-3} - 0.61 y_{t-4} + 0.48 y_{t-5} - 0.39 y_{t-6} \\ &\quad + 0.28 y_{t-7} + e_t^{(2)}, \ \textit{if}\ 2.373 < y_{t-2} < 3.154 \\ &= 2.323 + 1.530 y_{t-1} - 1.266 y_{t-2} + e_t^{(3)}, \ \textit{if}\ y_{t-2} \ge 3.154 \end{aligned}$$

2.3 Exponential Autoregressive (EAR) Models

EAR models and their extensions were studied by Ozaki and Oda (1978), Haggan and Ozaki (1981), and Lawrance and Lewis (1980).

The general form of the p-th order EAR models can be defined as :

$$f\left(y_{t-i},\ e_{t-j},\ i = 1,\dots,p,\ j = 1,\dots,q;\ \theta\right) = \delta + \sum_{i=1}^{p} \theta_i y_{t-i} \tag{2.9}$$

where $\theta_i = \alpha_i + \beta_i \exp\left(-\gamma y_{t-d}^2\right)$, $i = 1, \ldots, p$, $\gamma > 0$ is the smoothness parameter, d is the delay parameter, and y_{t-d} is the transition variable.

When $\gamma \to 0$ or ∞, the models become AR(p) models. When $\gamma < \infty$, the models display non-linear behavior. The coefficients of the variables switch smoothly with changes of y_{t-d}. The models can capture a situation in which the periods surrounding the turning points of a time series have similar dynamic structures whereas the middle ground can have different dynamics.

Haggan and Ozaki (1981) proposed the following estimation procedure for estimating the coefficients $\{\delta, \gamma, (\alpha_i, \beta_i, i = 1, \ldots, p)\}$ and order p for d = 1 in (2.9) by ordinary least squares:

I. First, the value of γ is fixed; then $\{\delta, (\alpha_i, \beta_i; i = 1, \ldots, p)\}$ can be estimated by ordinary least squares. The order of p is determined by minimizing AIC.

II. The above analysis is repeated for each γ and AIC is used to select the most suitable value of γ. The values of γ are usually chosen from a range such that $\exp\left(-\gamma y_{t-1}^2\right)$ is different from both zero and one for most values of y_{t-1}.

They then used the above procedure to estimate the Canadian lynx data[5] as follows:

$$\begin{aligned}
x_t = &\left[1.09 + 0.01\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-1} + \left[-0.28 - 0.49\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-2} \\
&+ \left[0.27 - 0.06\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-3} + \left[-0.45 + 0.30\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-4} \\
&+ \left[0.41 - 0.54\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-5} + \left[-0.36 + 0.61\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-6} \\
&+ \left[0.22 - 0.53\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-7} + \left[-0.10 + 0.30\exp\left(-3.89x_{t-1}^2\right)\right]x_{t-8}
\end{aligned}$$

$$+\left[0.22 - 0.18\exp\left(-3.89x_{t-1}^{2}\right)\right]x_{t-9} + \left[-0.07 + 0.18\exp\left(-3.89x_{t-1}^{2}\right)\right]x_{t-10}$$

$$+\left[-0.38 + 0.16\exp\left(-3.89x_{t-1}^{2}\right)\right]x_{t-11}$$

2.4 Smoothing Transition Autoregressive (STAR) Models

STAR models can be written as:

$$f\left(y_{t-i}, e_{t-j}, i = 1,\dots,p, j = 1,\dots,q; \theta\right)$$

$$= \delta_0 + \sum_{i=1}^{p} \delta_i y_{t-i} + \left(\theta_0 + \sum_{1=1}^{p} \theta_i y_{t-i}\right) F\left(y_{t-d}\right) \qquad (2.10)$$

where $F\left(y_{t-d}\right)$ is a continuous function which may be either even or odd, d is the delay parameter, and y_{t-d} is the transition variable. Bacon and Watts (1971) and Maddala (1977: 396) were early proponents of such models. Granger and Teräsvirta (1993) systematically described the functional form, specification, estimation and some application results of the STAR models. The STAR models can be used to describe the situation of smooth switch.

If we define

$$F\left(y_{t-d}\right) = \left\{1 + \exp\left[-\gamma\left(y_{t-d} - c\right)\right]\right\}^{-1} \qquad (2.11)$$

and $\gamma > 0$ is the smoothness parameter, we get logistic smoothing transition autoregressive (LSTAR) models.

LSTAR models are state-dependent autoregressive models with the parameter of y_{t-i} changing monotonically with changes of y_{t-d}. When $\gamma \to \infty$, $F\left(y_{t-d}\right)$ becomes a Heaviside indicator function: $F\left(y_{t-d}\right) = \begin{cases} 1 & if\ y_{t-d} \geq c \\ 0 & if\ y_{t-d} < c \end{cases}$, and (2.10) become TAR models. When $\gamma \to 0$, (2.10) becomes AR(p) models. LSTAR models describe situations where $y_{t-d} > c$ and $y_{t-d} < c$ may have rather different dynamics, and a transition from one to the other may be smooth.

If we define

$$F(y_{t-d}) = 1 - \exp\left[-\gamma(y_{t-d} - c)^2\right] \qquad (2.12)$$

and $\gamma > 0$ is the smoothness parameter, we get exponential smoothing transition autoregressive (ESTAR) models.

ESTAR models are state-dependent autoregressive models with the parameters of y_{t-i} changing symmetrically about c with changes of y_{t-d}. When $\gamma \to \infty$ or 0, $F(y_{t-d}) = 1$, (2.10) become AR(p) models. Applied to the modeling of business cycle indicators, ESTAR models describe situations where $y_{t-d} > c$ and $y_{t-d} < c$ have rather similar dynamic structures whereas the middle ground can have different dynamics.

According to Granger and Teräsvirta (1993), the specification of STAR consists of three steps:

I. Using the AIC or SBC to specify an AR(p) model and determining the possible set D for d.

II. Testing linearity for different values of the delay parameter d and, if it is rejected, simultaneously determining d.

The details are described as follows:

For a fixed d, linearity test against STAR consists of testing

$H_0 : \theta_{2i} = \theta_{3i} = \theta_{4i} = 0, i = 1, ..., p$

against H_1: H_0 is not valid in the artificial regression (2.13):

$$y_t = \theta_0 + \sum_{i=1}^{p} \theta_{1i} y_{t-i} + \sum_{i=1}^{p} \theta_{2i} y_{t-i} y_{t-d} + \sum_{i=1}^{p} \theta_{3i} y_{t-i} y_{t-d}^2 + \sum_{i=1}^{p} \theta_{4i} y_{t-i} y_{t-d}^3 + v_t \qquad (2.13)$$

The test is carried out for the range of $1 \le d \le D$ considered appropriate. If the linearity is rejected for more than one value of d, d is determined as $\hat{d}$ = argmin p(d) for $1 \le d \le D$, where p(d) is the p-value of the linearity test.

III. Choosing between LSTAR and ESTAR models using a sequence of tests of nested hypotheses.

The sequence of nested hypotheses to be tested is as follows:

$H_{04} : \theta_{4i} = 0, i = 1,\ldots, p$

$H_{03} : \theta_{3i} = 0 \mid \theta_{4i} = 0, i = 1,\ldots, p$

$H_{02} : \theta_{2i} = 0 \mid \theta_{3i} = \theta_{4i} = 0, i = 1,\ldots, p$

If the test of H_{03} has the smallest p-value, choose an ESTAR model; otherwise select a LSTAR model.

Once models have been chosen, the estimation can be done by non-linear least squares. Teräsvirta and Anderson(1992) used the technique above to model the indices of industrial production for 13 OECD countries and Europe, and the relationship between U.S. GNP and the U.S. Department of Commerce index of leading indicators. Superior in-sample performance was found. For example, they estimated the four-quarter differences, y_t, of the logarithm of U.S. industrial production index as the following LSTAR model[6]:

$$y_t = -0.021 + 0.35y_{t-1} + 0.24y_{t-3} - 1.03y_{t-4} + 0.33y_{t-9}$$
$$+ (0.021 + 1.61y_{t-1} - 0.57y_{t-2} - 0.24y_{t-3} 1.03y_{t-4} - 0.33y_{t-9}) *$$
$$(1 + \exp[-49 * 17.5(y_{t-3} - 0.0061)])^{-1} + \hat{e}_t$$

where the restrictions $\delta_i = \theta_i$, i = 0, 3, 4, 9, are imposed, suggested by the data.

2.5 Bilinear Autoregressive (BL) Models

The general form of bilinear models, BL (p, q, r, s), is:

$$y_t = \sum_{i=1}^{p} a_i y_{t-i} + e_t + \sum_{i=1}^{q} b_i e_{t-i} + \sum_{i=1}^{r} \sum_{j=1}^{s} c_{ij} y_{t-i} e_{t-j} \tag{2.14}$$

Bilinear models are natural extensions of ARMA models. From (2.14), we can see that bilinear models add the crossproducts of y_{t-i} and e_{t-j} to account for the non-linear character of the model: if all the c_{ij}s are zero, they reduce to ARMA models. Bilinear models have the property that they can approximate arbitrarily closely any reasonable non-linear relationship (Priestley, 1980).

Although bilinear models are natural extensions of ARMA processes, few economic applications appeared. Subba Rao and Gabr (1980) and Poskitt and Tremayne (1986) used economic time series to illustrate the methodology of specification, estimation, and forecast of bilinear models. Maravall (1983) used a bilinear model to estimate and forecast the Spanish currency series and found that bilinear models are particularly appropriate for series with sequences of outliers, during which periods a different "regime" seems to apply. Thus, the bilinear part of the model acts to smooth outliers; for the "normal" regime the bilinear part is dominated.

Subba Rao and Gabr (1984) proposed an algorithm for choosing the order of the BL (p, 0; r, s) as follows:

I. Choose a fixed integer γ, the maximum order to consider (γ should be greater than or equal to p*, the order of the best AR model).

II. For a $p \le \gamma$, fit a linear AR(p) model and let the corresponding residual variance be $\hat{\sigma}_e^2(AR)$.

III. Take the coefficients obtained from II as initial estimates of the autoregressive part of BL(p, 0; 1, 1), set $c_{11} = 0$, estimate the model, and calculate the corresponding $\hat{\sigma}_e^2$ and AIC for the fitted model.

IV. Fit the BL(p, 0; 1, 2) and BL(p, 0; 2, 1) by using the coefficients from III as initial values of the parameters and set the remaining parameters equal to zero. Calculate the corresponding $\hat{\sigma}_e^2$ and AIC for the fitted model.

V. Take the coefficients obtained from BL(p, 0; 1, 2) or BL(p, 0; 2, 1), whichever has smaller residual variance, as the initial values for fitting BL(p, 0; 2, 2). The procedure is continued, as shown in Fig. 2.1 for all possible combination (r, s), such that r, s < γ.

VI. Repeat all the steps II to V for p = 1, ..., γ and for each value of p. The procedure stops if the residual variance σ_e^2 increases as r and s increase.

VII. Choose the model which has the minimum AIC.

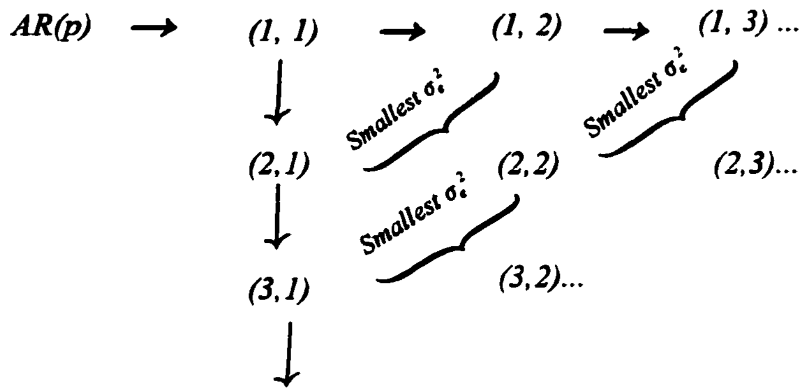


Figure 2.1 Algorithm to Choose Initial Values for Parameters of Bilinear Models

Source: Subba Rao and Gabr (1984), Fig. 5.1, page 177.

Subba Rao and Gabr (1984) maximized the likelihood function to estimate the parameters of bilinear models by the Newton-Raphson iteration method. They pointed out when fitted to a real time series, some of the coefficients for the full bilinear models of the form (2.14) may be "small", compared to other coefficients. Therefore, it is useful to see whether it is possible to fit a subset bilinear model to the data which leads to a parsimonious representation. The algorithm of choosing a subset bilinear model is:

I. Choose the best full AR(p) model.

II. Decide on the best subset AR model.

III. Add an term $y_{t-i}\, e_{t-j}$ such that AIC is minimized for all i, j $\leq$p. An extra term

$y_{t-v}\, e_{t-u}$ is added, where u, v $\leq$p and (u, v) $\neq$ (i, j). This is repeated until the minimum AIC is attained.

They used "subset" bilinear models to estimate and forecast the Wolfer sunspot data, Canadian lynx data and West German unemployment data. For example, the estimated bilinear model for the Wolfer Sunspot data[7] is:

$$x_t = 6.886 + 1.5012x_{t-1} - 0.767x_{t-2} + .1152x_{t-9} - 0.1458x_{t-2}\hat{e}_{t-1} + 0.006312x_{t-8}\hat{e}_{t-1}$$
$$- 0.007152x_{t-1}\hat{e}_{t-3} - 0.006047x_{t-4}\hat{e}_{t-3} + 0.003619x_{t-1}\hat{e}_{t-6} + 0.004334x_{t-2}\hat{e}_{t-4}$$
$$+ 0.001782x_{t-3}\hat{e}_{t-2} + \hat{e}_t$$

They then compared the results to ARMA and TAR models and found that bilinear models are better for estimation and forecast, compared with ARMA and TAR models. They concluded that subset bilinear models can form a useful class of non-linear models.

2.6 Generalized Autoregressive (GAR) Models

The general form of GAR models is:

$$y_t = \sum_i \sum_j a_{ij} y_{t-i}^j + \sum_p \sum_q \sum_u \sum_v b_{pquv} y_{t-u}^p y_{t-v}^q + \ldots + e_t \tag{2.15}$$

where i, j, p, q, u, v are integers which are greater or equal to 1 and higher-order crossproducts can be added. We can see that GAR models extensions of AR models by adding high-order lagged values and crossproducts. Mittnik (1991) introduced a GAR model as an autoregressive analogue of the discrete Volterra series, which can be interpreted as a non-linear moving-average representation of a generic non-linear process (see Priestley, 1988). Since GAR models are linear in their parameters, linear least square can be used to estimate the parameters. For example, Rothman (1998) estimated log-linear detrended U.S. unemployment rate as the following GAR model[8]:

$$U_t = 1.500\,U_{t-1} - 0.553\,U_{t-2} - 0.745\,U_{t-2}^3 + \hat{\varepsilon}_t$$

2.7 Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Models

The general form of GARCH (p, q) models is:

$$y_t = \sum_{t=1}^{m} \theta_{1i} y_{t-i} + e_t + \sum_{t=1}^{n} \theta_{2i} e_{t-i} \tag{2.16}$$

$$e_t = v_t \sqrt{h_t} \tag{2.17}$$

$$h_t = \gamma_0 + \sum_{t=1}^{q} \gamma_{1i} e_{t-i}^2 + \sum_{t=1}^{p} \gamma_{2i} h_{t-i} \tag{2.18}$$

where v_t is a normal white noise process with zero mean and variance of 1 and is independent of e_{t-1}, γ_0, $\{\gamma_{1i},\ i = 1, \ldots, q\}$ and $\{\gamma_{2j}, j = 1, \ldots, p\}$ are positive constants. Thus,

$$E(y_t|\psi_{t-1}) = \sum_{t=1}^{m} \theta_{1i} y_{t-i} + \sum_{t=1}^{n} \theta_{2i} e_{t-i} \tag{2.19}$$

$$Var(y_t|\psi_{t-1}) = h_t \tag{2.20}$$

where ψ_{t-1} is the information set at t. That is, y_t has nonconstant variances conditional on the past, but has constant unconditional variances. If p = 0, GARCH(p, q) models become ARCH(q) processes.

Engle (1982) introduced ARCH models to estimate the nonconstant conditional variance, described the maximum likelihood estimation of ARCH process and the Lagrange multiplier procedure to test the ARCH process. In the end, he applied a ARCH model to estimate the means and variances of inflation in the U.K. [9] as follows:

$$p_t = 0.0328 + 0.162 p_{t-1} + 0.264 p_{t-4} - 0.325 p_{t-5} - 0.0707(P-W)$$

$$h_t = 14*10^{-6} + 0.955\left(0.4e_{t-1}^2 + 0.3e_{t-2}^2 + 0.2e_{t-3}^2 + 0.1e_{t-4}^2\right)$$

He found that the ARCH effect is significant and the estimated variances are more realistic than that of OLS during the chaotic seventies. Bollerslev (1986) extended Engle's ARCH models to GARCH models. GARCH models are now extensively used for financial time series.

Notes

[1] See Tong (1983), pages 35-44, 77-82 and 77-79 for the arguments about asymmetric limit cycles, amplitude dependent frequencies, and jump resonances, respectively.

[2] See Tong (1983), pages 93-95 for the definition of ergodicity.

[3] Akaike information criterion (AIC) = T*ln(RSS) + 2*p; Schwartz Bayesian criterion (SBC) = T*ln(RSS) + p*ln(T), where RSS is the residual sum of squares, which equals to $\sum_{t=1}^{T}(y_t - \hat{y}_t)^2$, $\hat{y}_t$ is the fitted value of y_t, T is the number of usable observations and p is the number of estimated parameters. In this dissertation, all model orders are selected by the SBC, which chooses more parsimonious models than the AIC, given that the residuals do not have significant serial correlations. Both the AIC and SBC are reported for all selected models.

[4] See Tsay (1989), page 237.

[5] See page194, Haggan and Ozaki (1981).

[6] See Teräsvirta and Anderson (1992), page 125 for detail. In this paper, the STAR models were tried for both the term structure of interest rates and the spread of pork prices in U.S.. But no models with significant coefficients of this form were found.

[7] See Subba Rao and Gabr (1984), pages 196-203 for detail.

[8] See Rothman (1998), page 165 for detail.

[9] See Engle (1982), page 1002.

CHAPTER 3 A MONTE CARLO STUDY OF OVERFITTING AND FORECASTING TIME SERIES

3.1 Introduction

Overfitting means including too many regressors in the selected models or using more complicated non-linear models to estimate a linear or simple non-linear relationship. Overfitting may happen for model selection. For linear regression, as well known, when more regressors are added to the estimation, the fitting is usually improved. Thus, the problem of including too many regressors may happen. Many model selection criteria have been proposed to deal with this problem. For example, minimization of the AIC and/or SBC is often used to select models in practice. The AIC and SBC are the weighted sums of residual sum of squares and the number of regressors. So, when more regressors are added to regression, the AIC and SBC may increase even though residual sum of squares reduces. This tradeoff between residual sum of squares and the number of regressors can prevent adding too many regressors to regression to some extent. Usually, the SBC selects more parsimonious models than the AIC because the SBC punishes additional regressors to a larger extent than the AIC (From Note 3 in Chapter 2, the AIC punishes an additional parameter by 2, while the SBC punishes an additional parameter by ln(T), which is far larger than 2 in practice). When non-linearity is introduced, the problem of overfitting becomes more complicated. With more flexibility, non-linear models can do better for estimation than linear models by intuition. Does overfitting happen? Can the AIC and SBC select correct models? We do not know. At the same time, how overfitting affects out-of-sample forecast is not clear. Many think that overfit models should forecast poorly for out-of-sample forecast and thus, minimization of MSPE[1] can be used to select correct models. Is this the case? Can we use MSPE as a criterion to choose correct models? This problem is very important because when a set of linear and non-linear time series models are estimated for a given data set and non-linear models are better than the linear models by the criteria of the AIC and SBC, it is hard to conclude whether the dominance of non-linear models results from non-linearity of the data or overfitting and what is the relationship between overfitting and forecasting. In order to answer these questions, I use a Monte Carlo simulation in the univariate time series context.

3.2 Experiment Design

The basic idea of the Monte Carlo simulation is as follows: First, generate a series from a known AR model, estimate it by different linear and non-linear models and calculate the corresponding AIC and SBC. Then, use these models to forecast one-step ahead recursively and calculate the corresponding MSPE. Finally, the results of the AIC, SBC and MSPE are used to check how different models behave for in-sample estimation and out-of-sample forecast and the possibility of overfitting. I will generate the true AR(1) and AR(2) models to study it.

Also, a series from a known TAR model is generated to check the possibility of "underfitting", i.e., fitting a non-linear time series by a linear time series, and "misspecification", i.e., fitting a non-linear time series by other non-linear time series.

The detailed procedure is described as follows:

I. Generate a true AR(2) model: $y_t = \rho_0 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + e_t$, where e_t is an i.i.d. normal (0,1) process, $\{\rho_i, i = 0, 1, 2\}$ is a constant vector, t = 1, 2, ..., 100, and y_1 and y_2 are also drawn from a normal process with the true mean of $\rho_0/(1 - \rho_1 - \rho_2)$ and variance of 1.

II. Estimate y_t as the true AR(2) model, AR model, ARMA model, TAR model with consistent estimate of the threshold (called TAR-C model), M-TAR model with consistent estimate of the threshold (called M-TAR-C model), GAR model, bilinear model and EAR model. Then the AIC and SBC are calculated for each model. The true AR(2) model is the model with known order and it is treated as a benchmark for comparison. The AR, TAR-C, M-TAR-C, EAR and GAR models are the best models searching up to the third order. The ARMA model is the best model searching up to ARMA(2,1). The bilinear model is the best model searching up to BL(2,1; 2,1). Here, the TAR-C model is of form (2.6) and (2.8), and the M-TAR-C model is of form (2.7) and (2.8). The consistent thresholds are estimated by Chan's (1993) method. All estimated parameters of the selected models, except the true AR(2) model, are significant at the 5% level.

III. Forecast y_t for each model, using one-step ahead recursive forecast for period of t = 51, 52, ..., 100. That is, the series y_t, t = 1,...T, is used to forecast period t = T +1 and this process is repeated for T = 50, ..., 99. Then, the MSPE is calculated for each model.

IV. Repeat step I to III 1000 times.

V. Report the means of the AIC, SBC and MSPE for each model in Table 3.1 and the distributions of the AIC, SBC, MSPE for some selected parameter vectors in Figure 3.1 through 3.12. The means and distributions are used to check the average levels and variances of the AIC, SBC and MSPE, respectively.

VI. Repeat step I to V for each parameter vector $\{\rho_i, i = 0, 1, 2\}$. In this study, The vector is chosen to be {1, 1.2, -0.5}, {1, 0.5, 0.4}, {1, 0.9, 0}, {1, 0.5, 0} and {1, 0, 0.9}. The distributions of the AIC, SBC and MSPE of the parameter vector of {1, 1.2, -0.5}, {1, 0.5, 0.4}, {1, 0, 0.9}and {1, 0.5, 0}are reported.

The above steps are repeated for the true TAR models (3.1):

$$y_t = \begin{cases} \rho_{10} + \rho_{11} y_{t-1} + e_t \ \ if\ y_{t-1} \geq \tau \\ \rho_{20} + \rho_{21} y_{t-1} + e_t \ \ if\ y_{t-1} < \tau \end{cases} \tag{3.1}$$

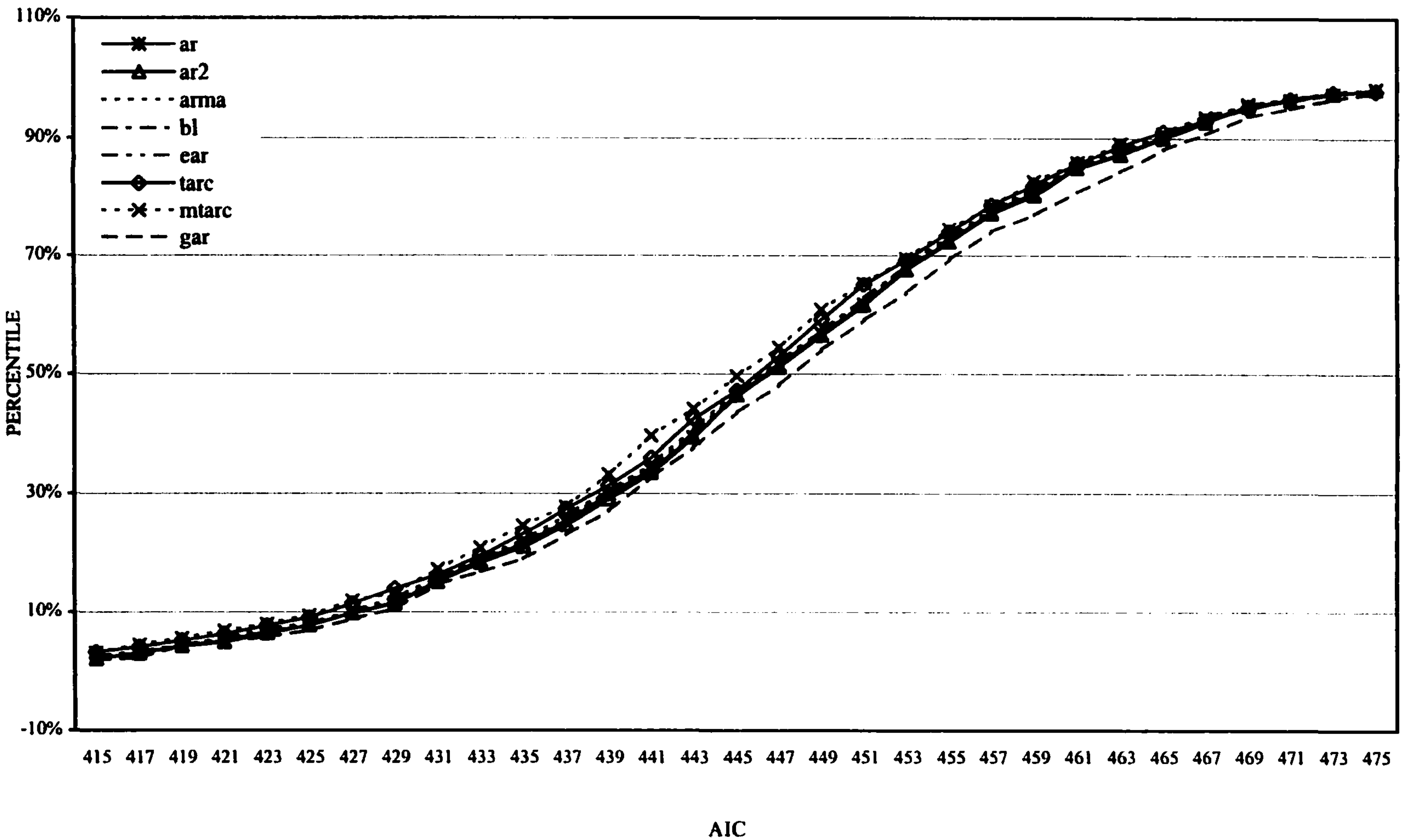
where $\{\rho_{10}, \rho_{11}, \rho_{20}, \rho_{21}, \tau\}$ is the parameter vector, e_t is an i.i.d. normal (0,1) process. In this study, the vector is chosen to be {0.5, 0.95, 0.9, 0.9, 9.5}, {0.3, 0.95, 3.2, 0.6, 7.2}, {0.4, 0.95, 4.2, 0.3, 7}, {0.4, 0.8, 1.2, 0.6, 2.5}, and {1.2, 0.8, 1.4, 0.3, 4}. The threshold is chosen to be in the interval of the two long-run equilibriums of the two regimes. For the vector of

Table 3.1 The Means of the AIC, SBC and MSPE for the True AR(2) Models

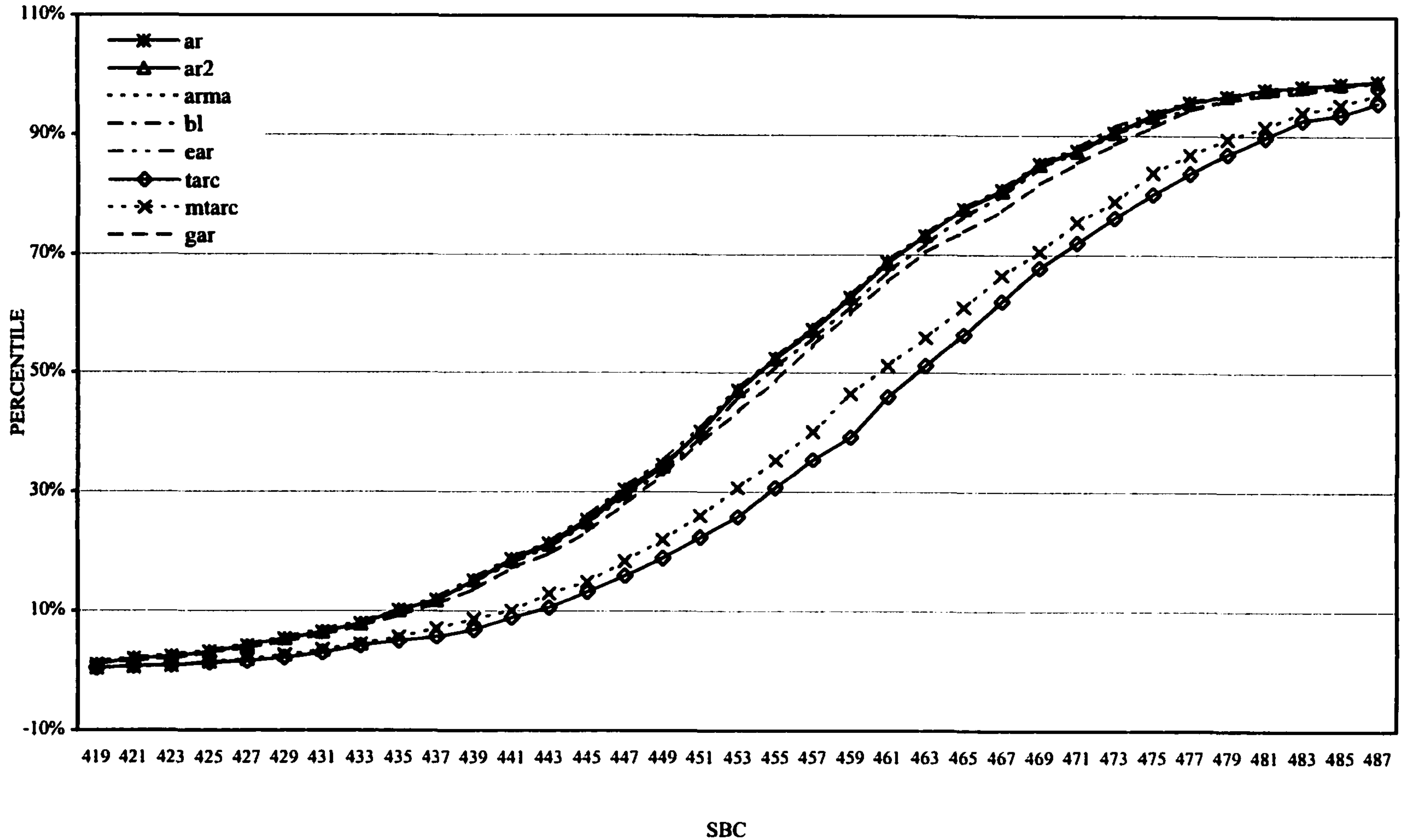
ρ_0	1.0	1.0	1.0	1.0	1.0
ρ_1	0.5	1.2	0.0	0.9	0.5
ρ_2	0.4	-0.5	0.9	0.0	0.0
AR1_AIC	NA	NA	NA	444.2697	444.4369
AR2_AIC	445.5328	446.2637	444.8585	NA	NA
ARMA_AIC	445.4386	446.0124	444.5904	445.1879	444.0585
BL_AIC	445.6017	446.0132	444.5625	445.7982	444.3319
AR_AIC	445.6361	446.1329	444.6688	444.0751	444.0712
EAR_AIC	445.6199	445.7208	443.8898	443.4854	443.7558
GAR_AIC	444.4124	447.3273	443.8698	443.0844	443.2822
TARC_AIC	442.9117	445.2952	442.0580	441.8941	442.1763
MTARC_AIC	442.6889[a]	444.8783	441.9651	441.9085	442.2259
AR2_SBC	453.2569	453.9878	450.0080	449.4191	449.5863
ARMA_SBC	452.2976	453.9271	449.5004	450.5768	449.7615
BL_SBC	453.0503	454.4274	449.9231	451.7123	450.7172
AR_SBC	452.4848	453.9136	449.4681	448.9980	449.4498
EAR_SBC	452.4763	453.6509	448.7998	448.5344	449.2683
GAR_SBC	451.3873	455.0282	448.8493	448.0923	448.9517
TARC_SBC	455.8985	462.0489	454.3626	453.0477	453.8475
MTARC_SBC	455.3307	460.2725	454.1924	453.7624	454.6335
AR2_MSPE	1.0664	1.0572	1.0440	1.0497	1.0371
ARMA_MSPE	1.1125	**1.0719**	1.0769	1.0985	1.0533
BL_MSPE	**1.1086**[b]	1.0726	**1.0737**	1.1077	**1.0524**
AR_MSPE	1.1208	1.0749	1.0796	**1.0841**	1.0558
EAR_MSPE	1.1220	1.0978	1.0841	1.0885	1.0793
GAR_MSPE	1.1222	1.2152	1.0969	1.1006	1.0904
TARC_MSPE	1.2804	1.2824	1.2874	1.2454	1.2169
MTARC_MSPE	1.2695	1.2389	1.2772	1.2334	1.2044

[a] The underlined numbers are the smallest among all the models for the AIC, SBC and MSPE for each parameter vector.

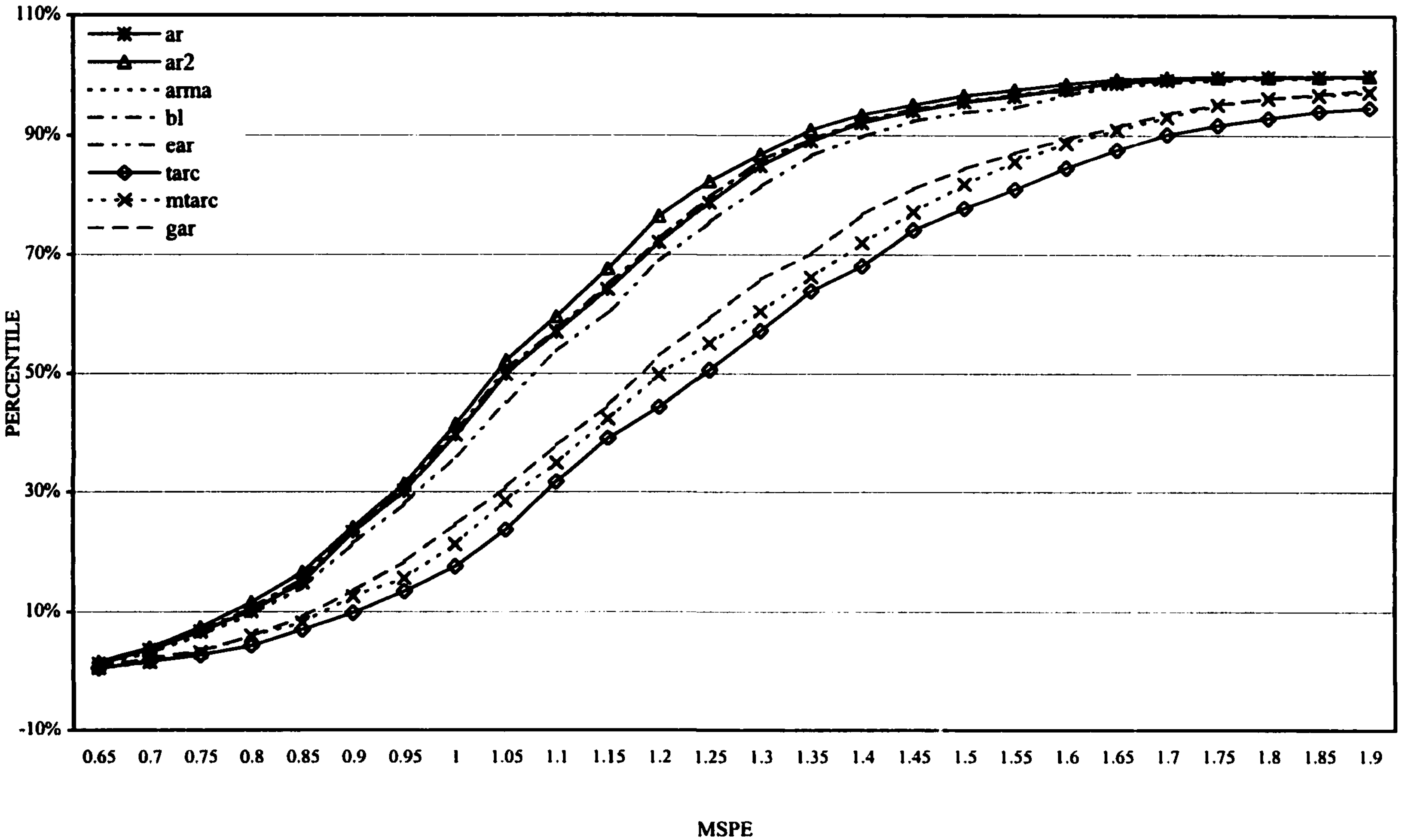
[b] The bold numbers are the smallest MSPEs except the true AR(2) models.



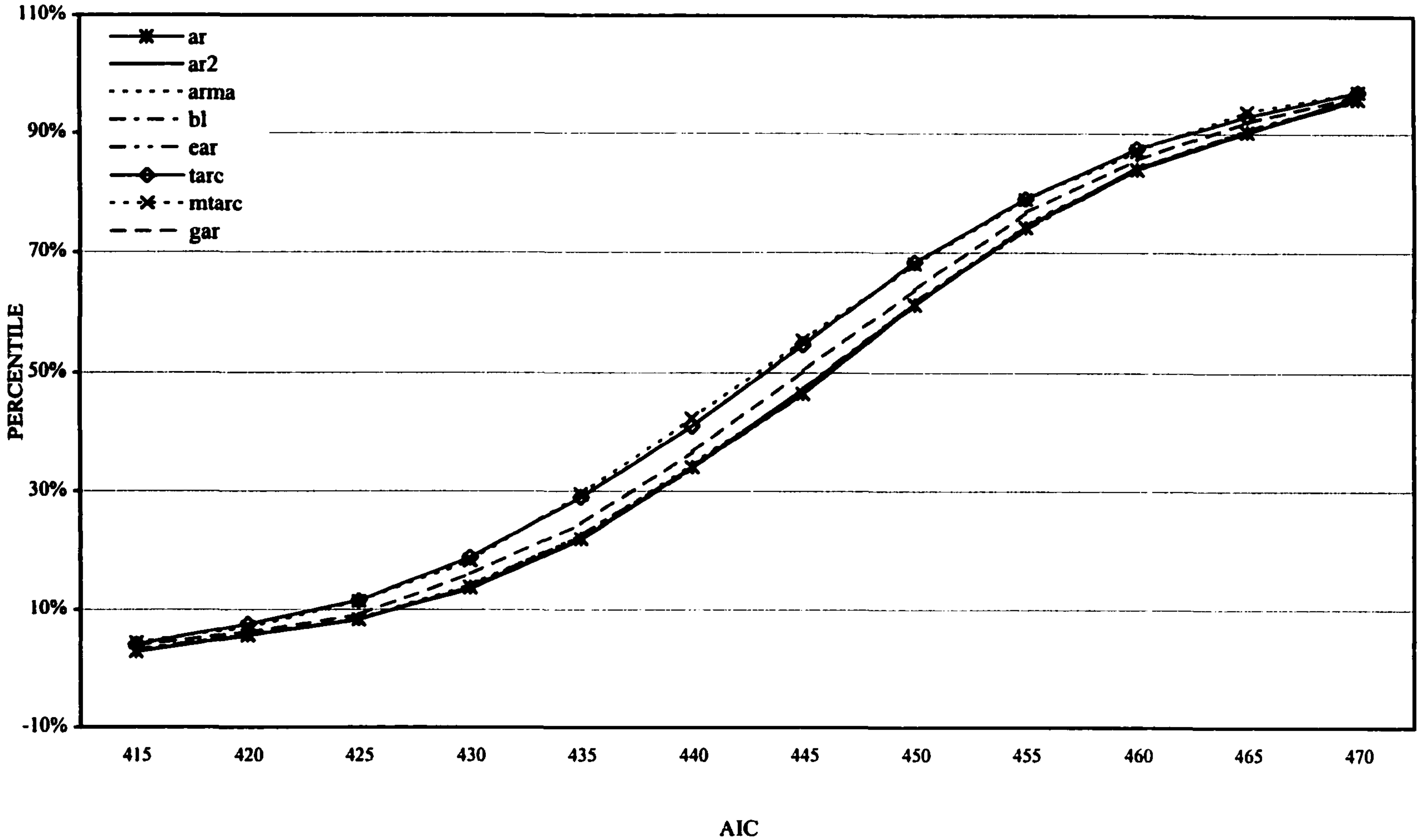
Finger 3.1 Distributions of the AIC for the True AR(2) Model:
y(t) = 1 + 1.2*y(t-1) - 0.5*y(t-2) + e(t)



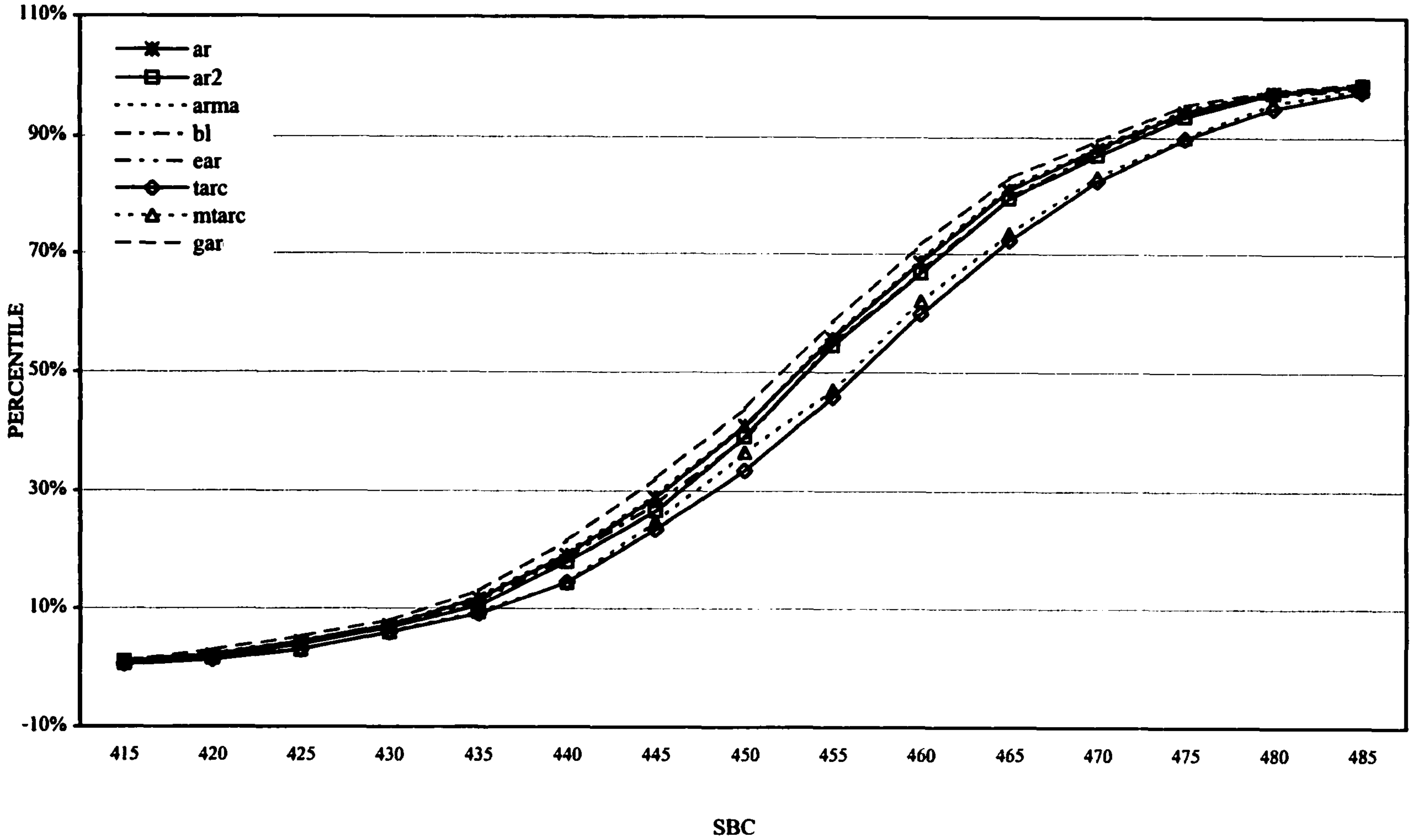
Finger 3.2 Distributions of the SBC for the True AR(2) Model:
y(t) = 1 + 1.2*y(t-1) - 0.5*y(t-2) + e(t)



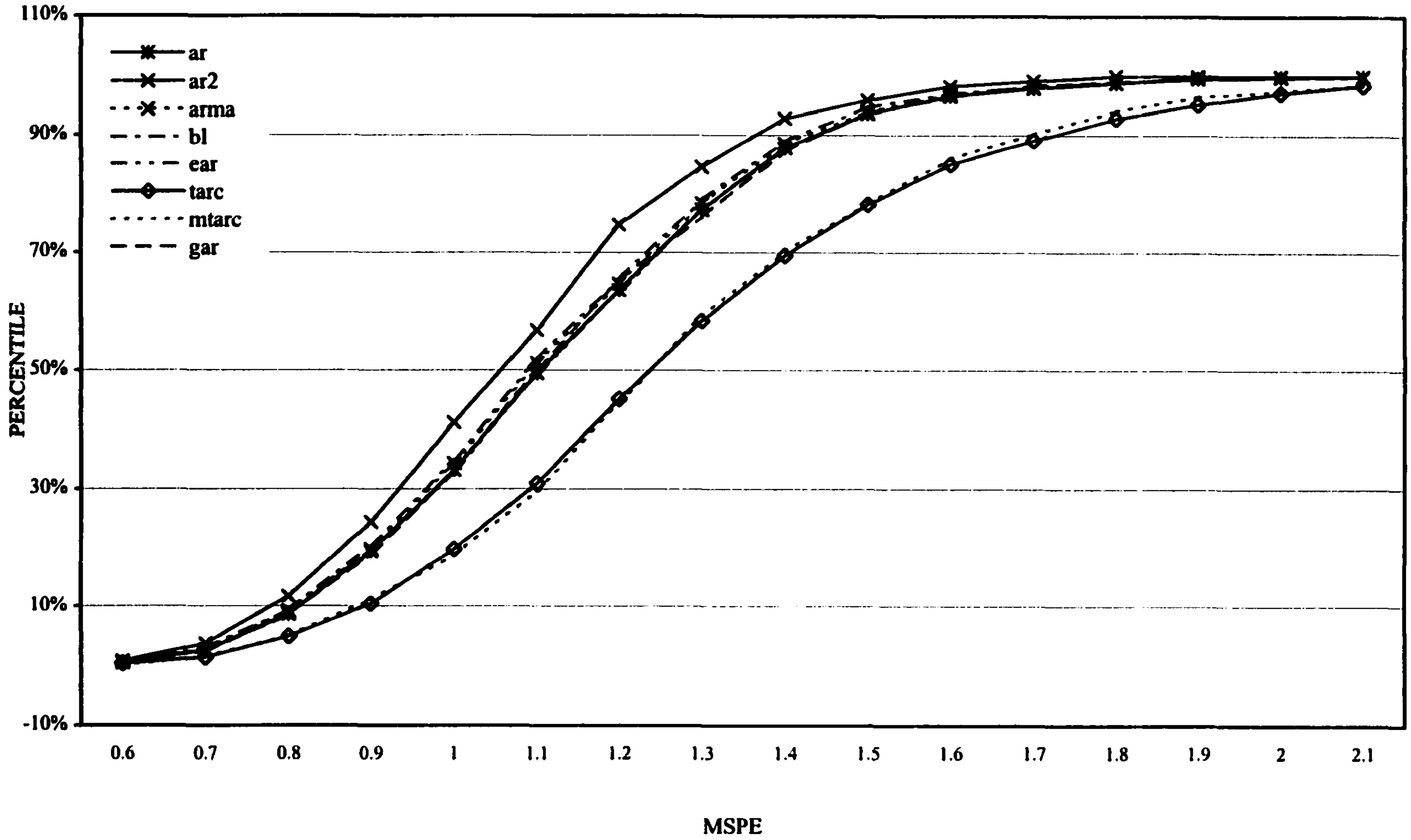
Finger 3.3 Distributions of the MSPE for the True AR(2) Model:
y(t) = 1 + 1.2*y(t-1) - 0.5*y(t-2) + e(t)



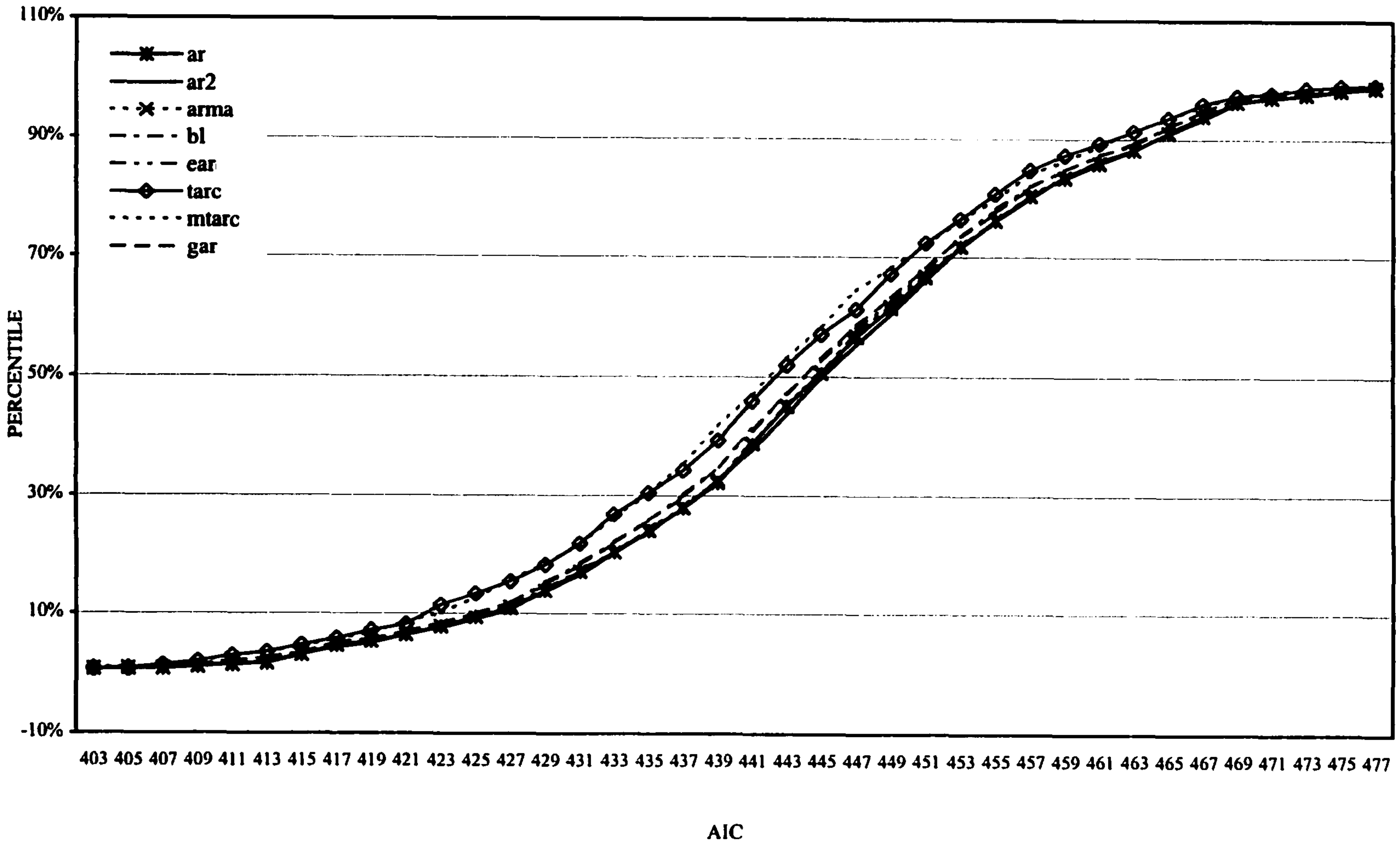
Finger 3.4 Distributions of the AIC for the True AR(2) Model:
y(t) = 1 + 0.5*y(t-1) + 0.4*y(t-2) + e(t)



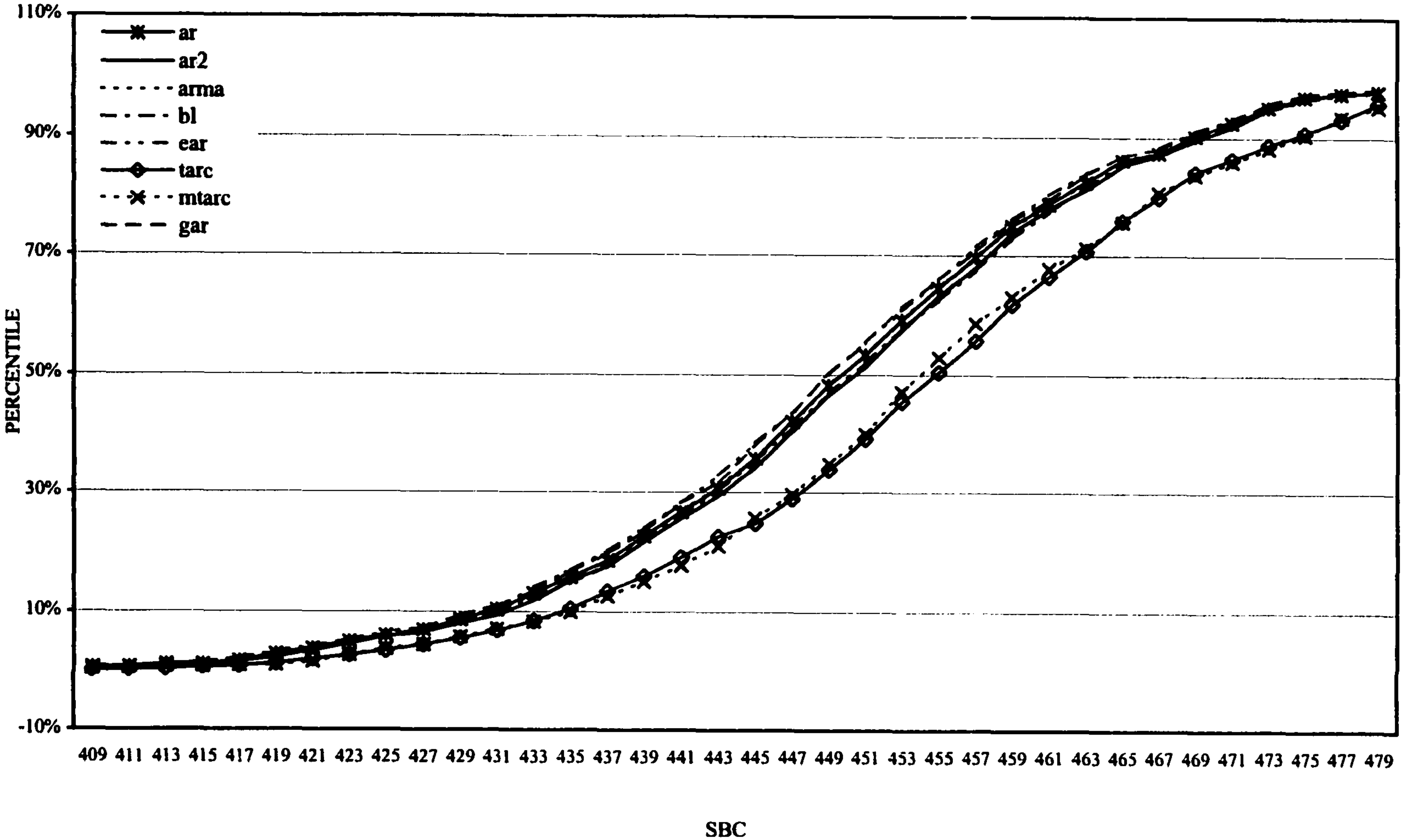
Finger 3.5 Distributions of the SBC for the True AR(2) Model:
$y(t) = 1 + 0.5*y(t-1) + 0.4*y(t-2) + e(t)$



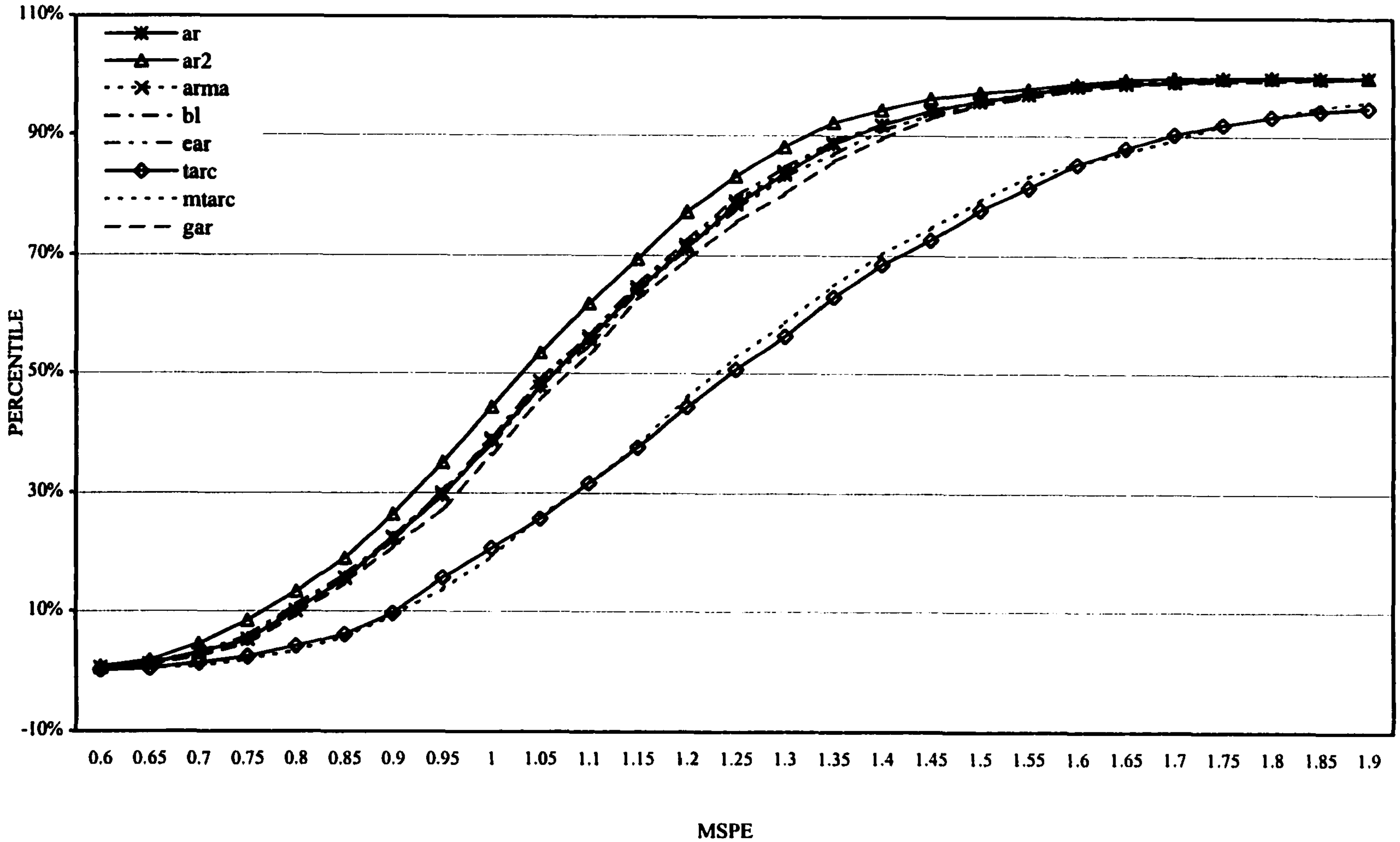
Finger 3.6 Distributions of the MSPE for the True AR(2) Model:
y(t) = 1 + 0.5*y(t-1) + 0.4*y(t-2) + e(t)



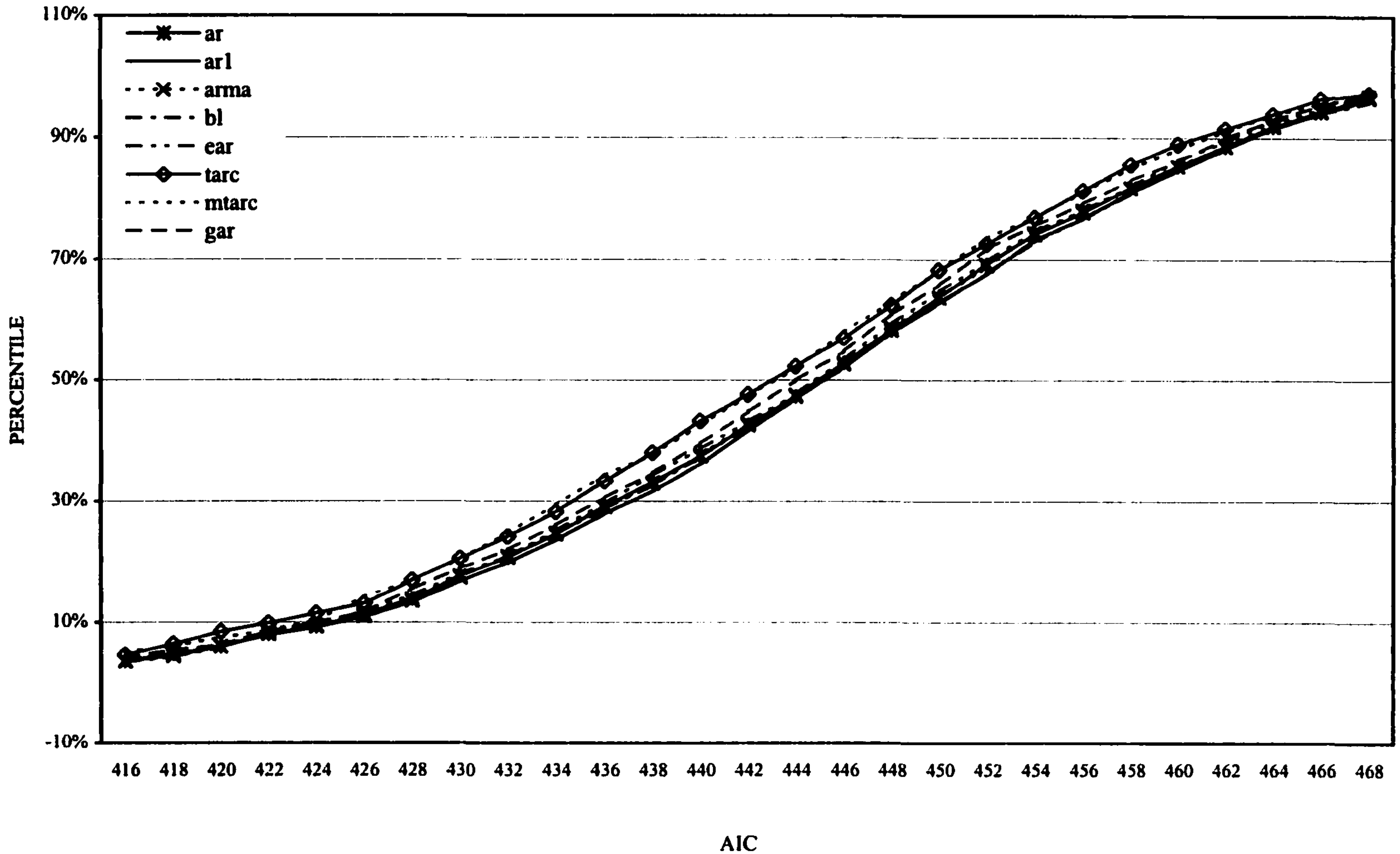
Finger 3.7 Distributions of the AIC for the AR(2) True Model:
y(t) = 1 + 0*y(t-1) + 0.9*y(t-2) + e(t)



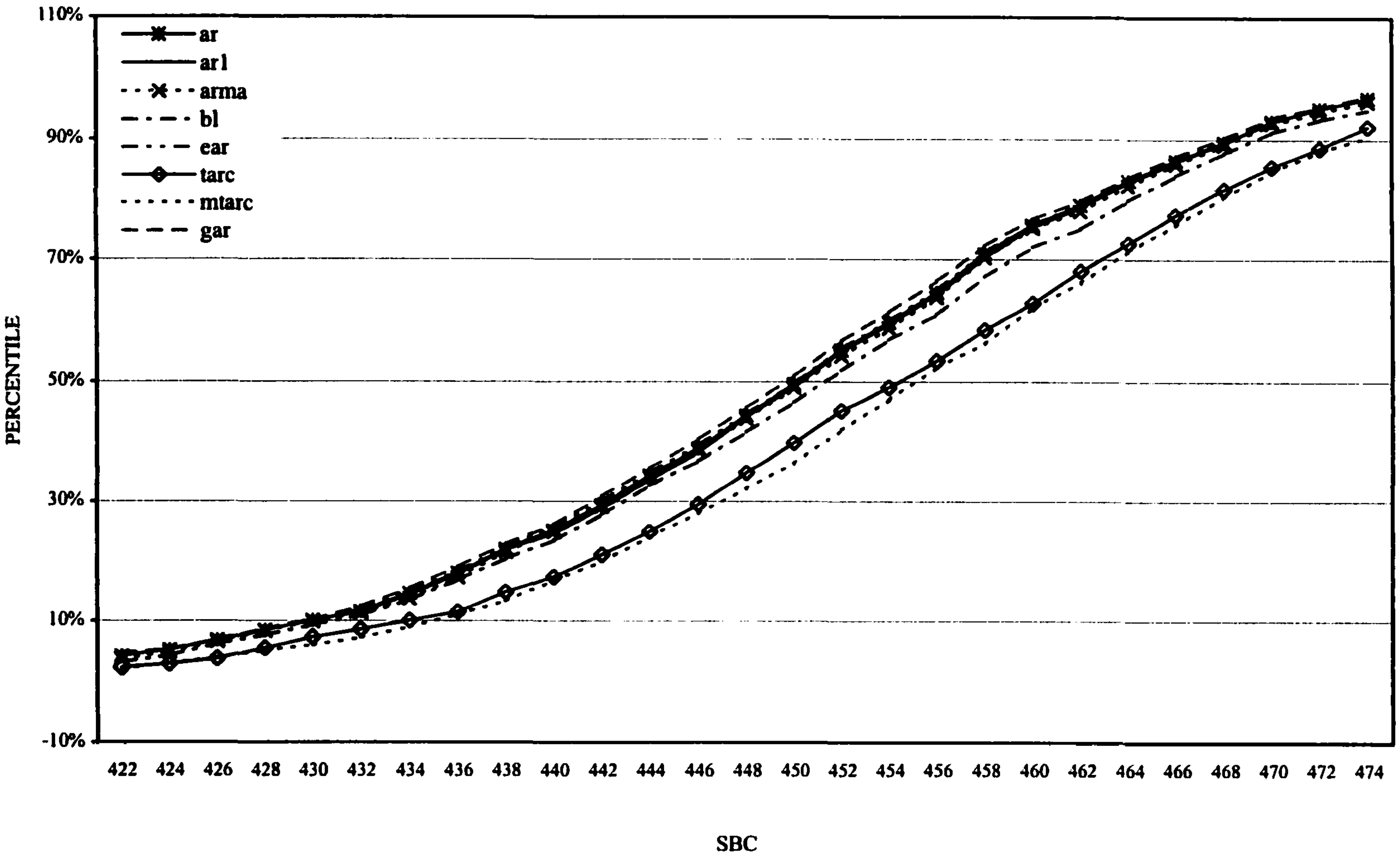
Finger 3.8 Distributions of the SBC for the True AR(2) Model:
$y(t) = 1 + 0*y(t-1) + 0.9*y(t-2) + e(t)$



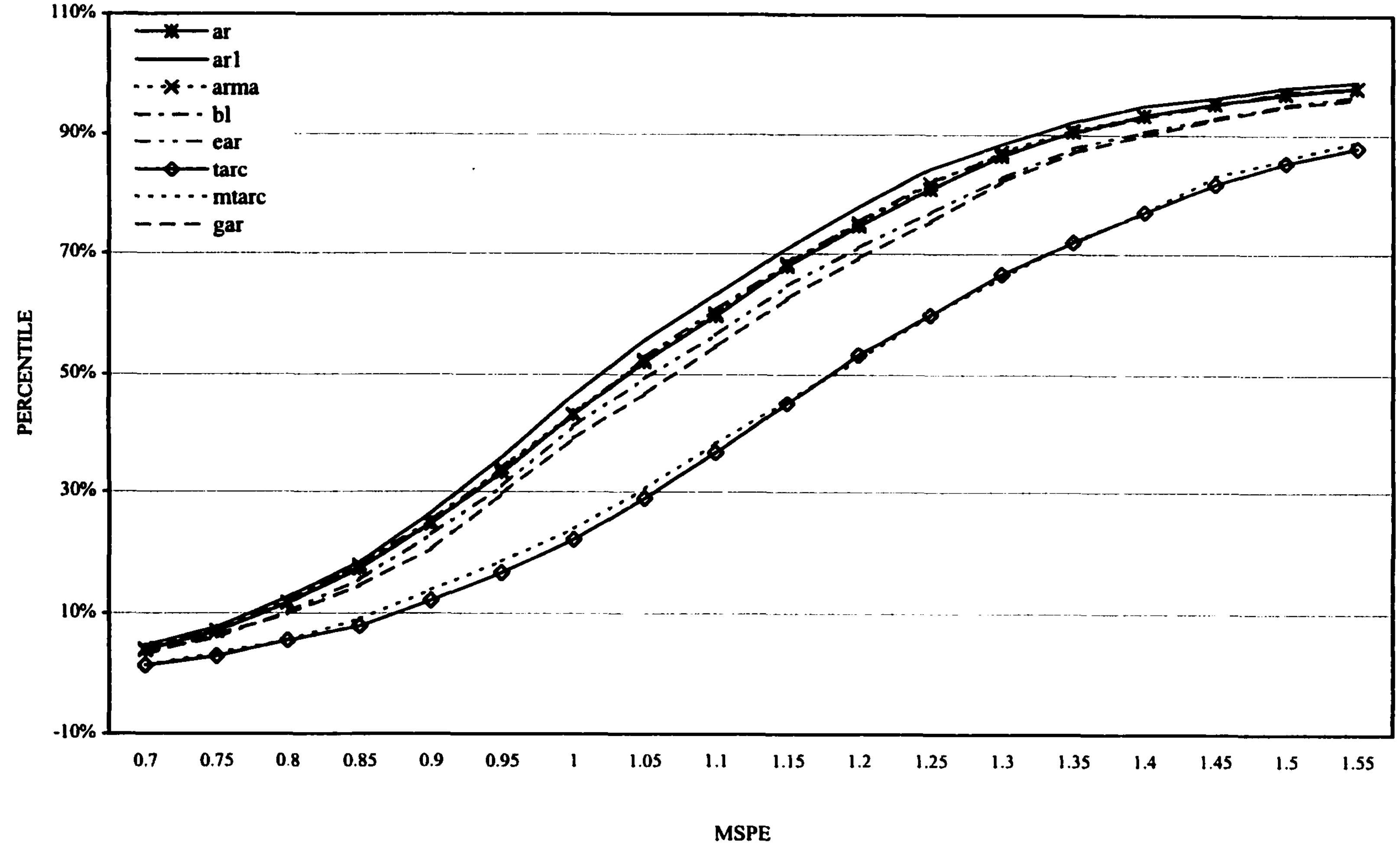
Finger 3.9 Distributions of the MSPE for the True AR(2) Model:
y(t) = 1 + 0*y(t-1) + 0.9*y(t-2) + e(t)



Finger 3.10 Distributions of the AIC for the AR(2) True Model:
$y(t) = 1 + 0.5*y(t-1) + 0*y(t-2) + e(t)$



Finger 3.11 Distributions of the SBC for the AR(2) True Model:
y(t) = 1 + 0.5*y(t-1) + 0*y(t-2) + e(t)



Finger 3.12 Distributions of the MSPE for the AR(2) True Model:
$y(t) = 1 + 0.5*y(t-1) + 0*y(t-2) + e(t)$

{0.5, 0.95, 0.9, 0.9, 9.5}, the AR(1) processes above and below the threshold have long-run equilibriums of 10 and 9, respectively. Also, the difference of the persistence between the two regimes is very small. For the vector of {0.3, 0.95, 3.2, 0.6, 7.2}, the AR(1) processes above and below the threshold have long-run equilibriums of 6 and 8, respectively. Also, the difference of the persistence between the two regimes is median. For the vector of {0.4, 0.95, 4.2, 0.3, 7}, the AR(1) processes above and below the threshold have long-run equilibriums of 8 and 6, respectively. Also, the difference of the persistence between the two regimes is large. For the vector of {0.4, 0.8, 1.2, 0.6, 2.5}, the AR(1) processes above and below the threshold have long-run equilibriums of 2 and 3, respectively. Also, the difference of the persistence between the two regimes is median. For the vector of {1.2, 0.8, 1.4, 0.3, 4}, the AR(1) processes above and below the threshold have long-run equilibriums of 6 and 2, respectively. Also, the difference of the persistence between the two regimes is large. In this simulation, the two long-run equilibrium in the two regimes are chosen to be not very far apart from each other. Otherwise, the series will be mostly concentrated on one of the two regimes. This kind of threshold models can be illustrated in Figure 3.13, where μ^+ is the long-run equilibrium for the AR process above the threshold, μ^- is the long-run equilibrium for the AR process below the threshold.

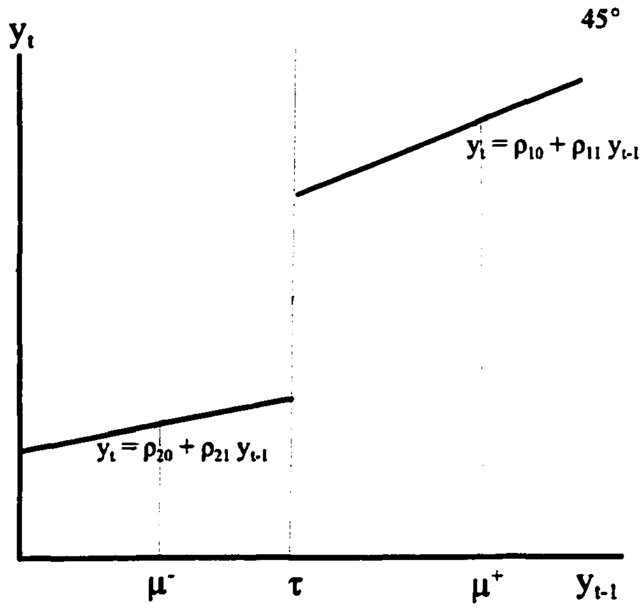


Figure 3.13 The Threshold Autoregressive Model with Two Distinct Long-run Equilibriums

In fact, the TAR models with only one long-run equilibrium (see Figure 3.14) are also tried, but since the observations are attracted to the only long-run equilibrium, the observations on the lower side of the threshold are very rare. Thus, the results are not very good because we have too few observations to estimate the model on the lower side of the threshold accurately. In this experiment, I ignore this case.

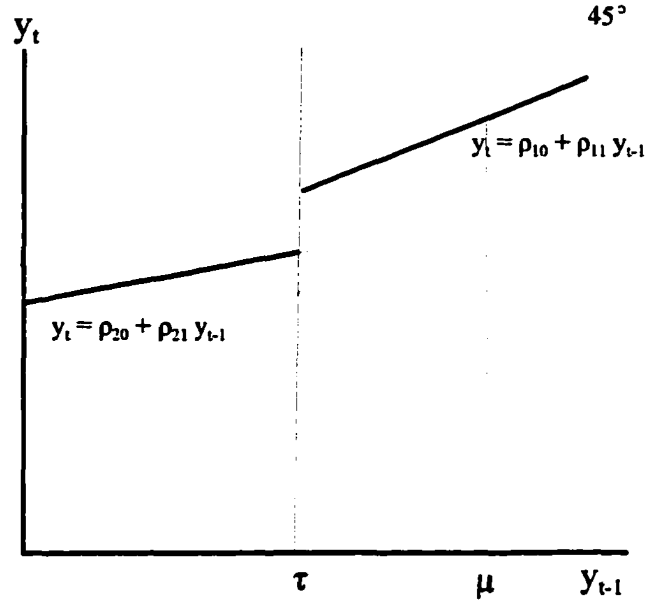


Figure 3.14 The Threshold Autoregressive Model with Only One Long-run Equilibrium

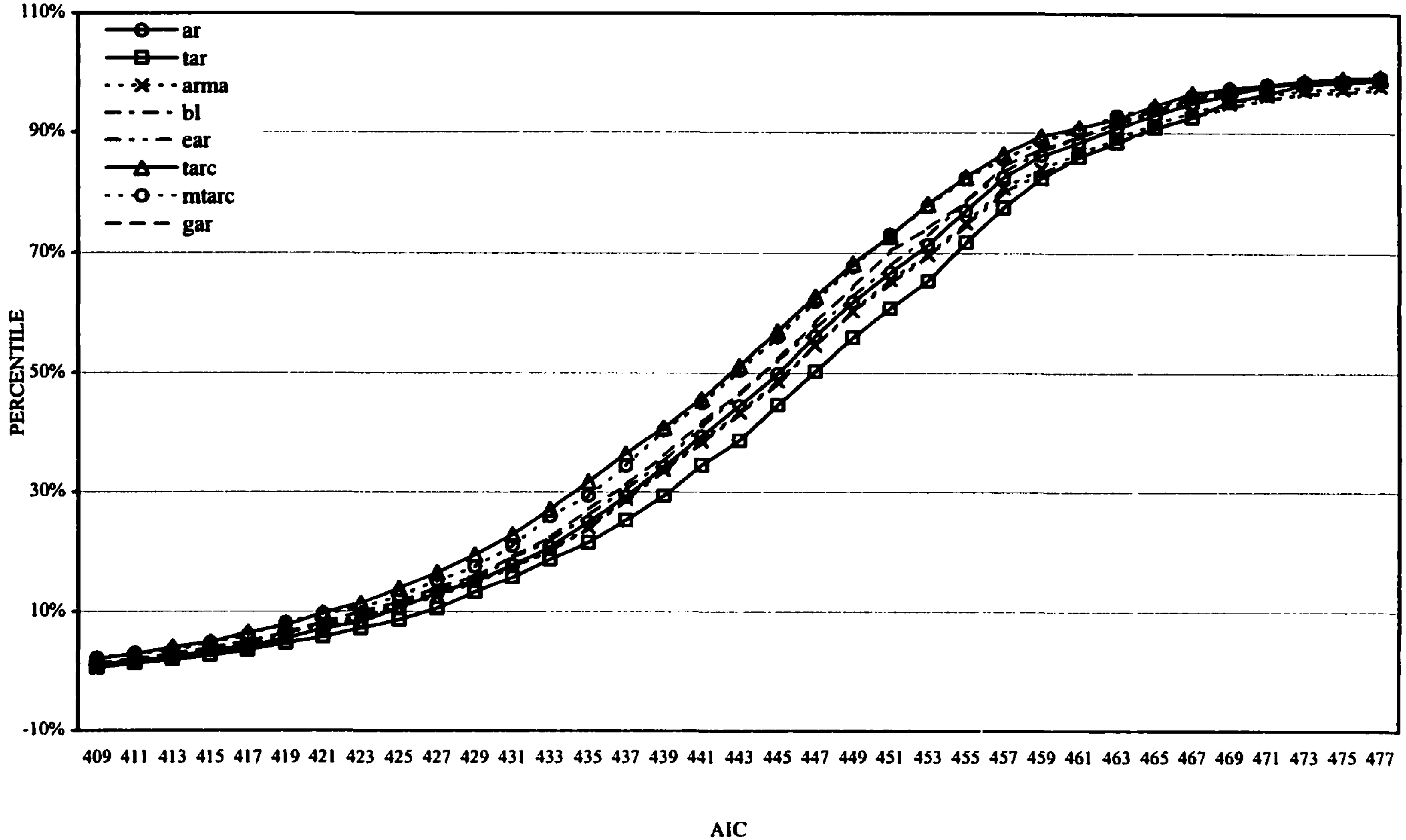
For the TAR process, each series is estimated by the true TAR model, AR model, ARMA model, TAR-C model, M-TAR-C, GAR model, bilinear model and EAR model. I estimate the true TAR model with known threshold and order of 1 and treat it as a benchmark for comparison. All other models are selected by the same rule as above. Then the AIC and SBC are calculated for each model. The selected models are used to forecast one-step ahead recursively and the MSPE is calculated. The means of the AIC, SBC, MSPE for each model are reported in Table 3.2 and the distributions of the AIC, SBC, MSPE for each parameter vector of {0.5, 0.95, 0.9, 0.9, 9.5}, {0.4, 0.95, 4.2, 0.3, 7} and {1.2, 0.8, 1.4, 0.3, 4} are shown by Figure 3.15 though Figure 3.23.

Table 3.2 The Means of the AIC, SBC and MSPE for the True TAR Models

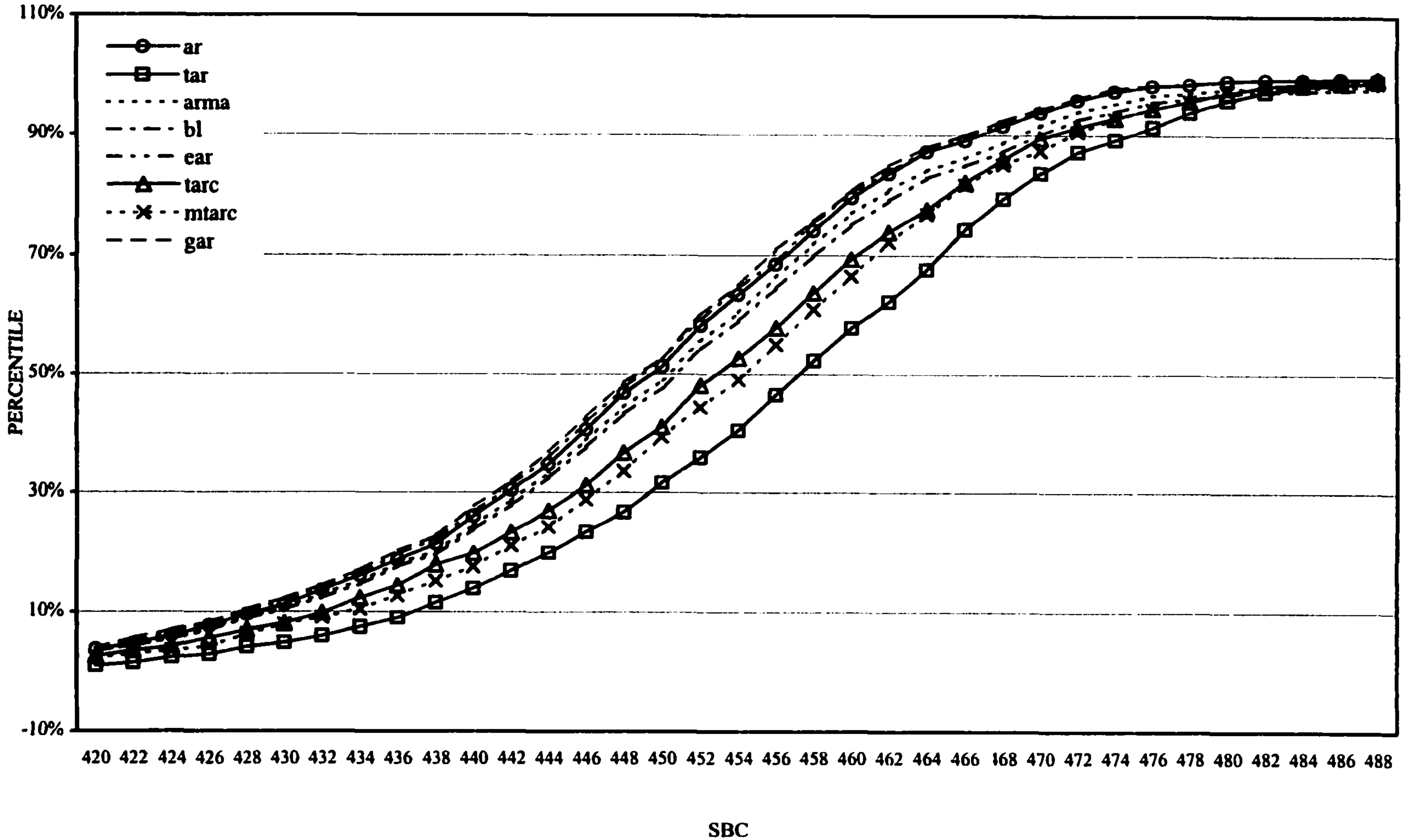
ρ_{10}	0.50	0.30	0.40	0.40	1.20
ρ_{11}	0.95	0.95	0.95	0.80	0.80
ρ_{20}	0.90	3.20	4.20	1.20	1.40
ρ_{21}	0.90	0.60	0.30	0.60	0.30
τ	9.50	7.20	7.00	2.50	4.00
TAR_AIC	446.0258	446.1787	446.0276	445.6452	449.3015
ARMA_AIC	445.1133	447.8633	452.5516	444.7521	464.6882
BL_AIC	445.5999	448.3674	452.5603	445.0513	464.8412
AR_AIC	444.0961	447.0731	452.2506	444.5751	464.3222
EAR_AIC	443.5589	445.9150	450.5393	443.9089	462.2699
GAR_AIC	443.1783	445.9794	450.4730	443.5974	462.0072
TARC_AIC	441.4928 [a]	441.7442	442.6870	441.4967	449.1756
MTARC_AIC	441.8919	444.8020	448.6649	442.7876	460.6419
TAR_SBC	456.3246	456.4775	456.3265	455.9440	459.6004
ARMA_SBC	449.9461	453.5251	457.8941	450.4577	469.7784
BL_SBC	450.8858	454.9998	459.1310	451.5653	471.2084
AR_SBC	448.4500	452.3152	457.1658	449.9408	469.0236
EAR_SBC	448.0852	451.5382	455.9462	449.5604	467.3730
GAR_SBC	447.7690	451.2653	455.6018	449.1356	467.2828
TARC_SBC	452.2628	452.7459	453.1687	452.8589	460.4863
MTARC_SBC	453.2310	456.8362	460.0426	455.2080	471.9192
TAR_MSPE	1.0932	1.0750	1.0822	1.0656	1.0983
ARMA_MSPE	1.0867	1.1122	1.1727	1.0573	1.2937
BL_MSPE	1.0980	1.1172	1.1710	1.0577	1.2910
AR_MSPE	**1.0687** [b]	**1.1014**	1.1697	**1.0565**	1.2929
EAR_MSPE	1.0707	1.1108	**1.1640**	1.0800	1.2807
GAR_MSPE	1.0845	1.1212	1.1771	1.0868	1.3060
TARC_MSPE	1.2366	1.2117	1.2156	1.2042	**1.2558**
MTARC_MSPE	1.2272	1.2638	1.3385	1.2163	1.4461

[a] The underlined numbers are the smallest among all the models for the AIC, SBC and MSPE for each parameter vector.

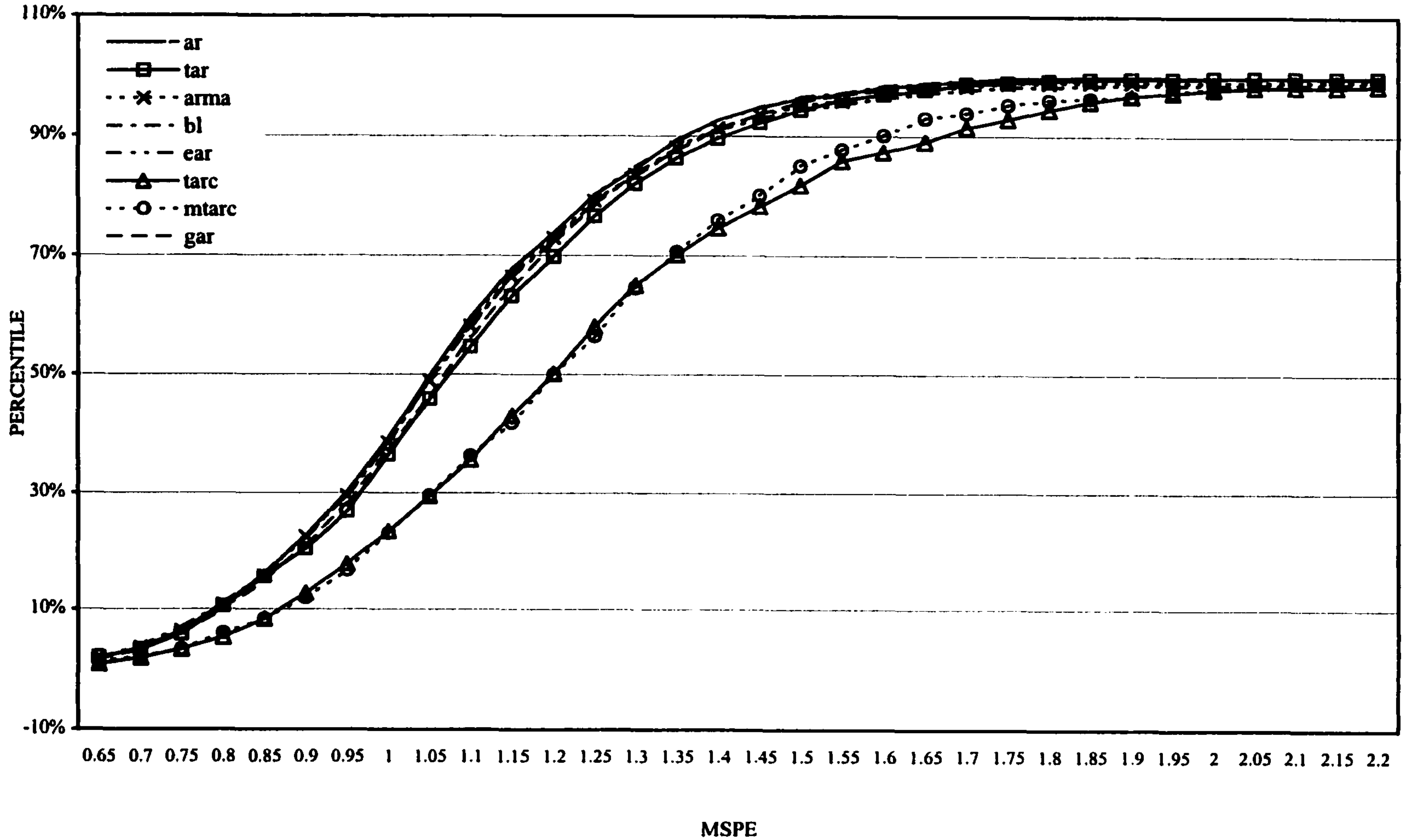
[b] The bold numbers are the smallest MSPEs except the true TAR models.



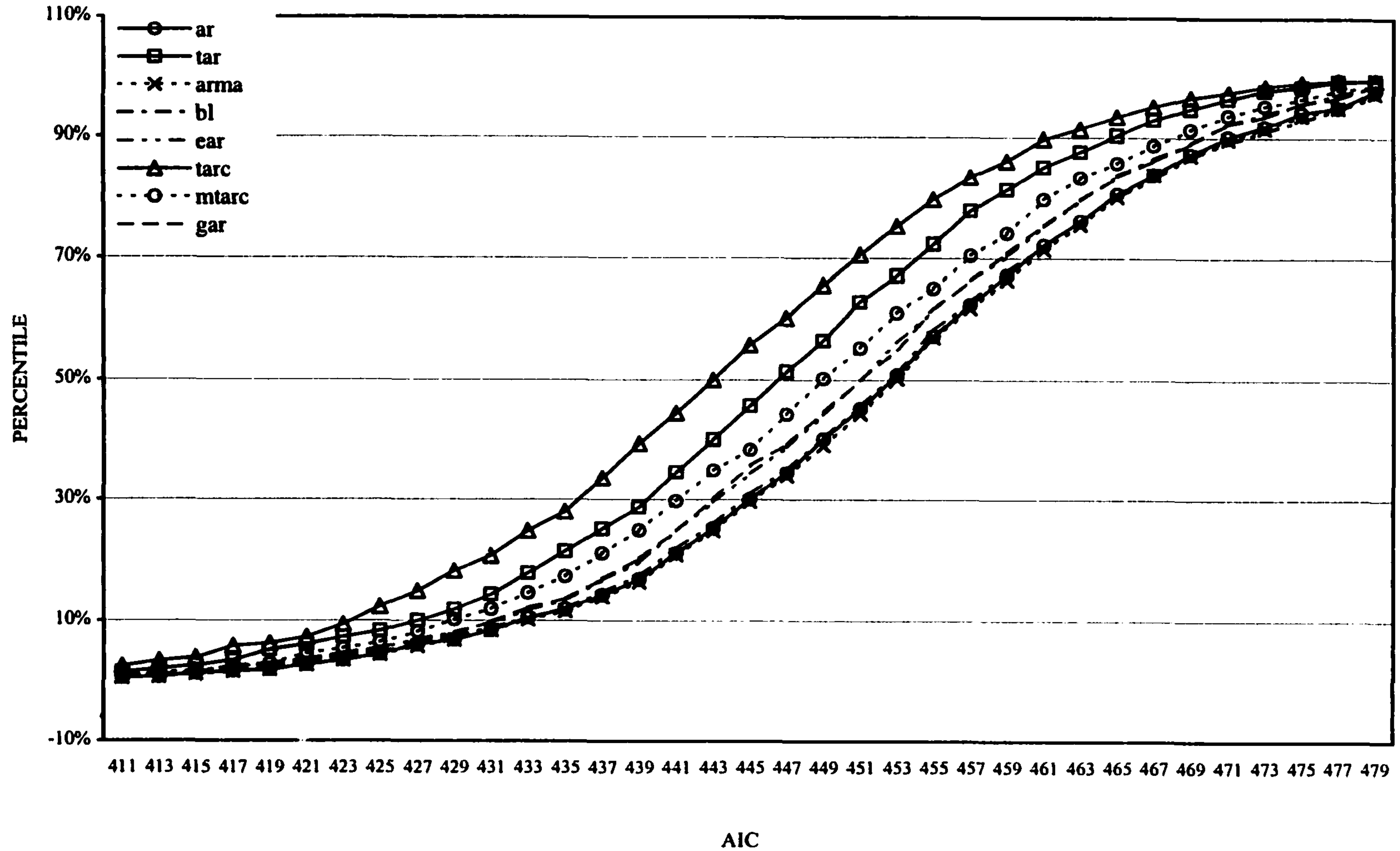
Finger 3.15 Distributions of the AIC for the True Threshold Model: y(t) = 0.5 + 0.95*y(t-1) + e(t), if y(t-1)>=9.5, y(t) = 0.9 + 0.9*y(t-1) + e(t), if y(t-1) < 9.5



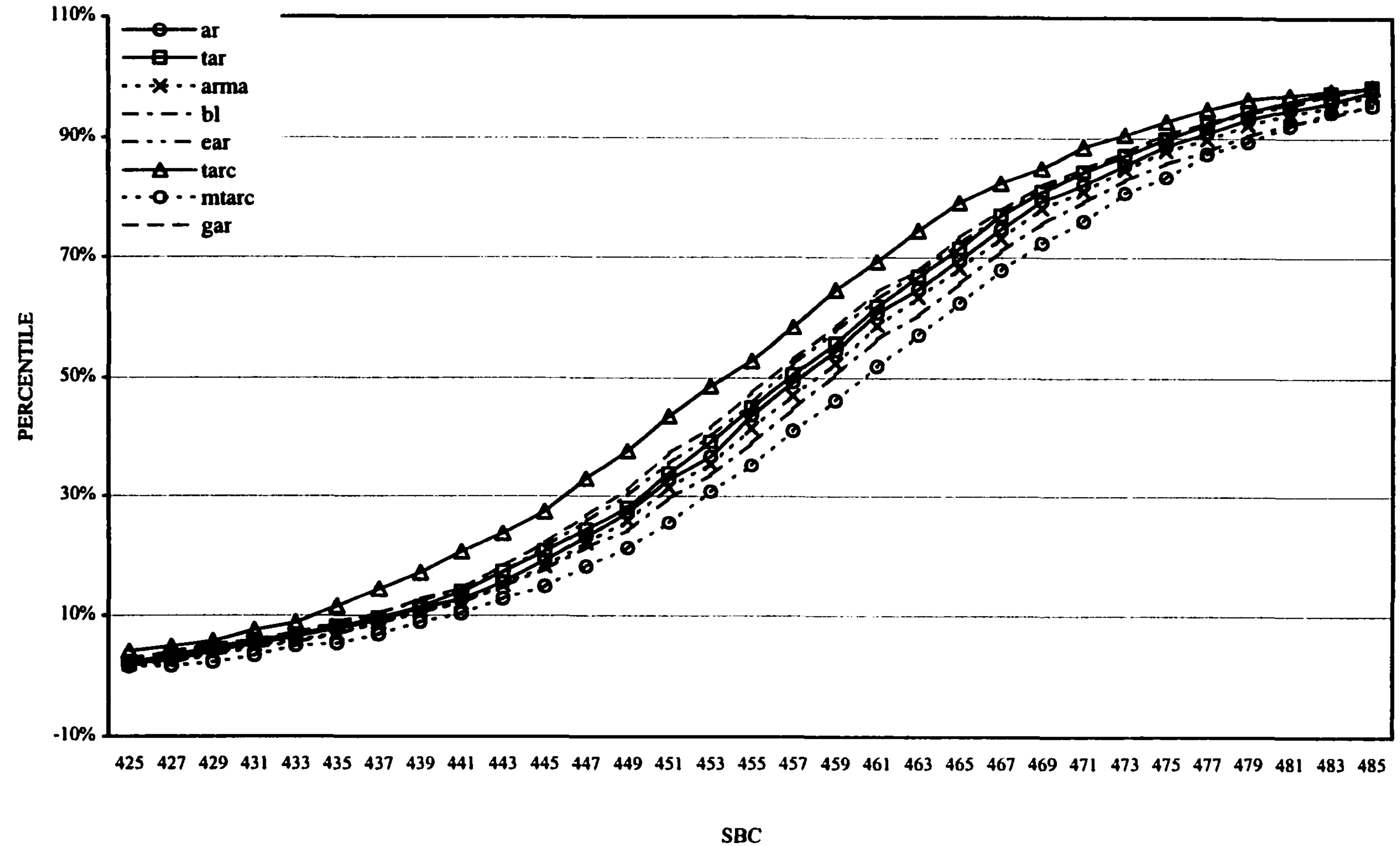
Finger 3.16 Distributions of the SBC for the True Threshold Model: y(t) = 0.5 + 0.95*y(t-1) + e(t), if y(t-1)>=9.5, y(t) = 0.9 + 0.9*y(t-1) + e(t), if y(t-1) < 9.5



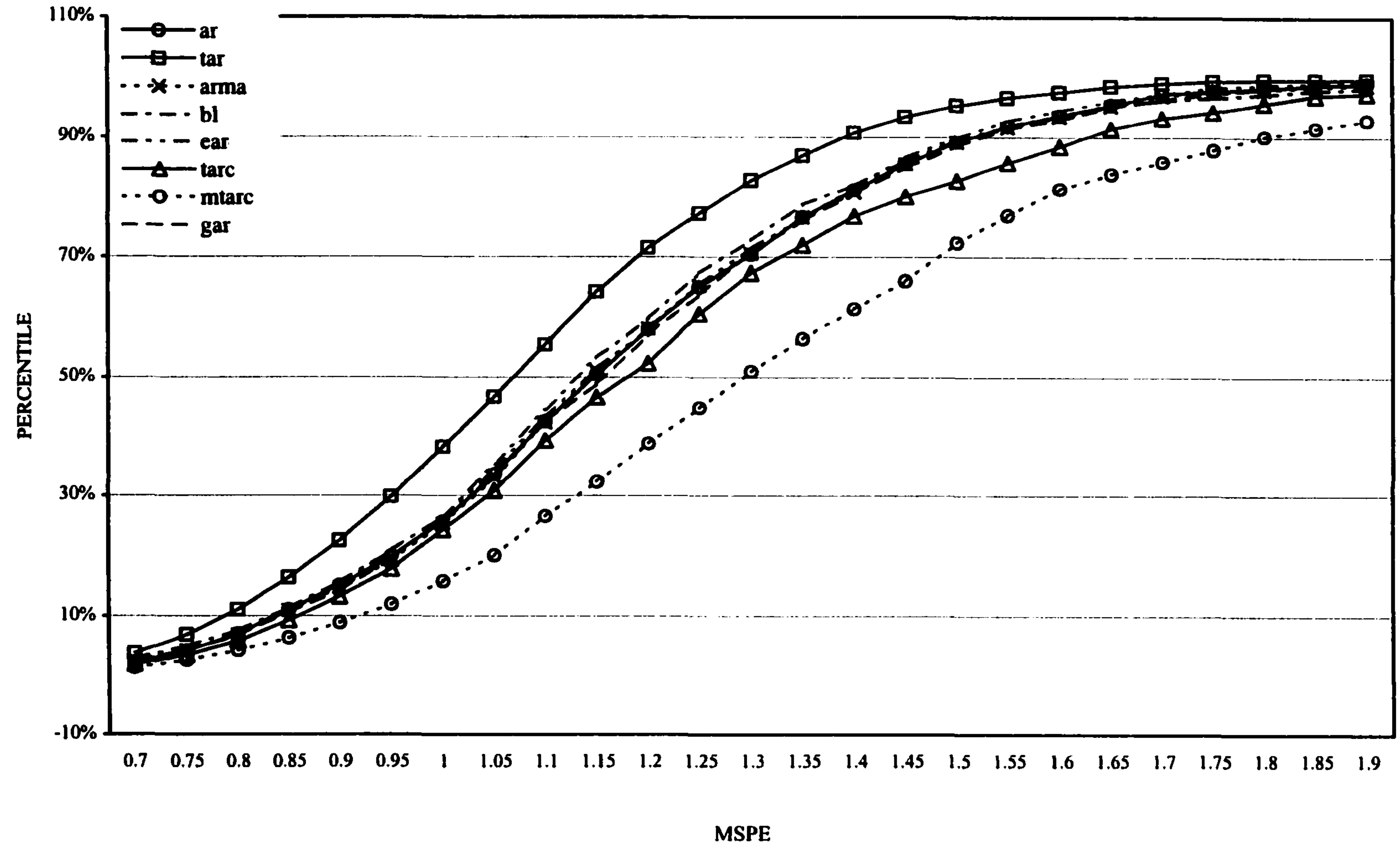
Finger 3.17 Distributions of the MSPE for the True Threshold Model: $y(t) = 0.5 + 0.95*y(t-1) + e(t)$, if $y(t-1) >= 9.5$, $y(t) = 0.9 + 0.9*y(t-1) + e(t)$, if $y(t-1) < 9.5$



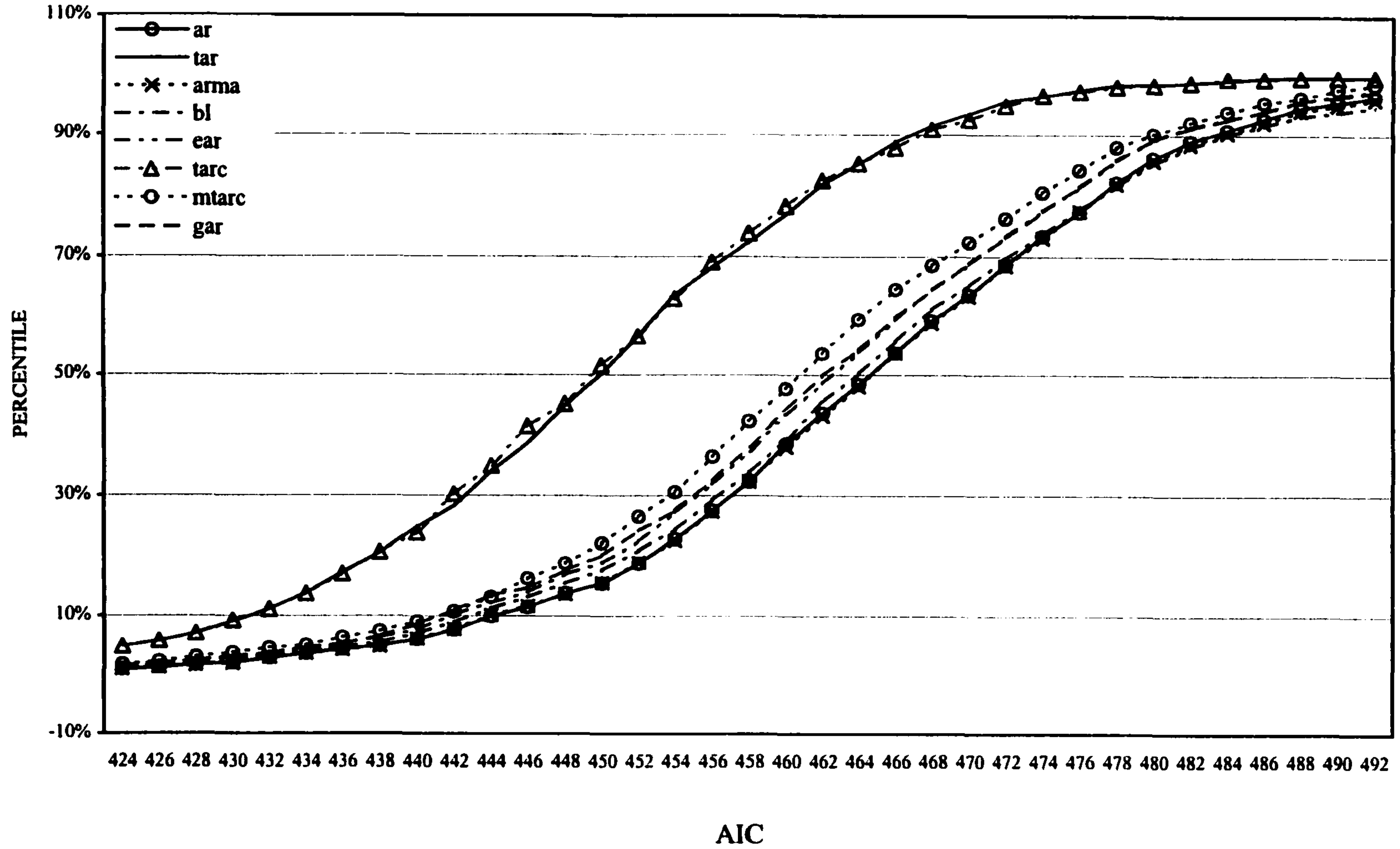
Finger 3.18 Distributions of the AIC for the True Threshold Model:
y(t) = 0.4 + 0.95*y(t-1) + e(t), if y(t-1) >= 7, y(t) = 4.2 + 0.3*y(t-1) + e(t), if y(t-1) < 7



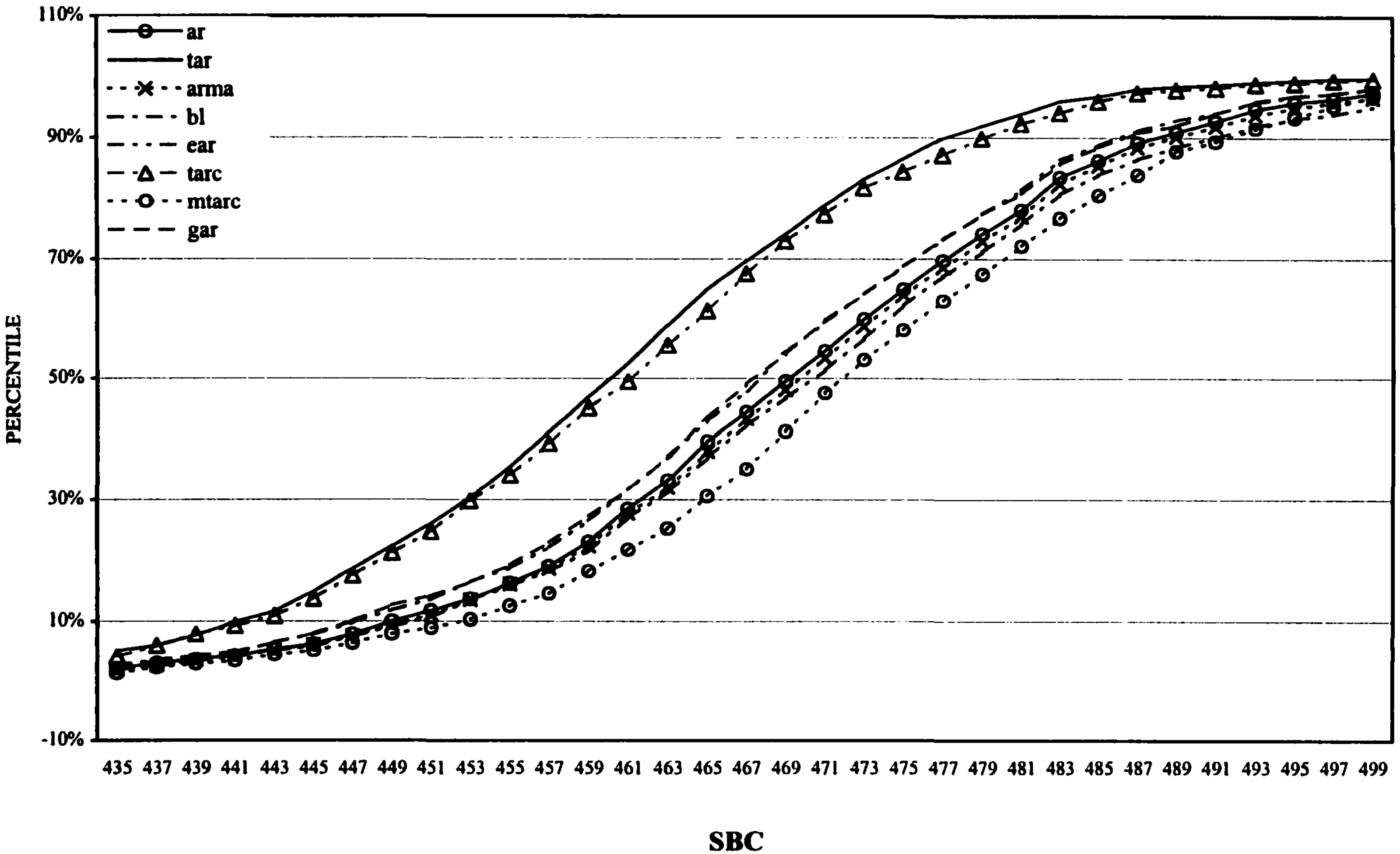
Finger 3.19 Distributions of the SBC for the True Threshold Model:
y(t) = 0.4 + 0.95*y(t-1) + e(t), if y(t-1) >= 7, y(t) = 4.2 + 0.3*y(t-1) + e(t), if y(t-1) < 7



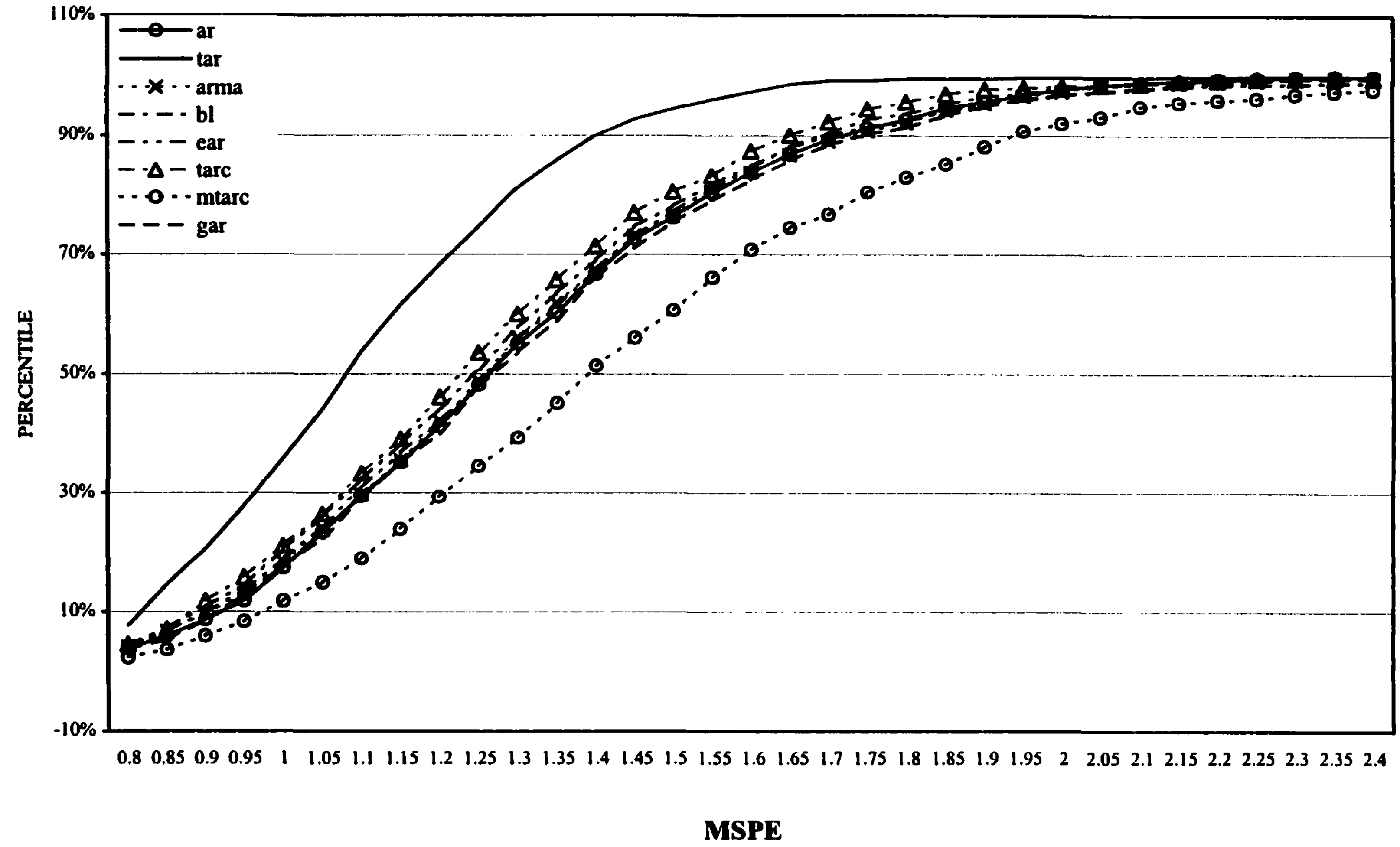
Finger 3.20 Distributions of the MSPE for the True Threshold Model:
y(t) = 0.4 + 0.95*y(t-1) + e(t), if y(t-1) >= 7, y(t) = 4.2 + 0.3*y(t-1) + e(t), if y(t-1) < 7



Finger 3.21 Distributions of the AIC for the True Threshold Model:
y(t) = 1.2 + 0.8*y(t-1) + e(t), if y(t-1) >= 4, y(t) = 1.4 + 0.3*y(t-1) + e(t), if y(t-1) < 4



Finger 3.22 Distributions of the SBC for the True Threshold Model:
y(t) = 1.2 + 0.8*y(t-1) + e(t), if y(t-1) >= 4, y(t) = 1.4 + 0.3*y(t-1) + e(t), if y(t-1) < 4



Finger 3.23 Distributions of the MSPE for the True Threshold Model:
y(t) = 1.2 + 0.8*y(t-1) + e(t), if y(t-1) >= 4, y(t) = 1.4 + 0.3*y(t-1) + e(t), if y(t-1) < 4

3.3 Results Report

3.3.1 The True AR(2) Models: $y_t = \rho_0 + \rho_1 y_{t-1} + \rho_2 y_{t-2} + e_t$

From Table 3.1, the true AR(2) models have relatively large AIC and SBC in all models. The TAR-C and M-TAR-C models have smaller AIC than all other models, which indicates that these models can reduce residual variances substantially. The GAR, EAR and AR models have smaller SBC than all other models and have relatively small AIC too. However, since the SBC punishes an additional parameter to a larger extent than the AIC, the TAR-C and M-TAR-C models, which usually estimate more parameters than other models, have larger SBC than all other models. The distributions of the AIC and SBC indicate the same results. Thus, since the AR and ARMA models can have smaller AIC and SBC than the true AR(2) models and the non-linear models can beat the linear models in the sense of reducing the AIC and SBC, the true AR(2) models can be dominated by other linear and non-linear models, which means that overfitting is very likely to happen for the AR processes if the AIC and SBC are used to select models.

From the means of the MSPE, the true AR(2) models have the smallest MSPE among all the models and the TAR-C and M-TAR-C models have larger MSPE than all other models. The ARMA, AR and bilinear models have relatively small MSPE. This means that for the out-of-sample forecast, the true AR(2) models can predict the best. The distributions of the MSPE are consistent with these results. Thus, the MSPE is a good criterion to choose correct models if the series is an AR process, but the AIC and SBC can not correctly choose models. If we don't know the true order of the AR process, we hope to select the correct type of models, i.e., the AR models. However, none of the AIC, SBC and MSPE can select the AR process because either the bilinear models or ARMA models have smaller MSPE than the AR models in most cases. The AR models forecast the best in only one case.

3.3.2 The True TAR models: $y_t = \begin{cases} \rho_{10} + \rho_{11} y_{t-1} + e_t & \text{if } y_{t-1} \geq \tau \\ \rho_{20} + \rho_{21} y_{t-1} + e_t & \text{if } y_{t-1} < \tau \end{cases}$

From Table 3.2, the TAR-C models have the smallest AIC among all the models, which means that TAR models with consistent estimate of threshold can reduce residual variance a lot. The M-TAR-C models have relatively small AIC too. When the difference of the persistence between the two regimes is small or median, the AR, ARMA, EAR and GAR usually have smaller AIC than the true TAR models with known threshold, which means that underfitting and misspecification are very likely to happen. When the difference of the persistence between the two regimes is large enough, the true TAR models have smaller AIC than other models except the TAR-C models, which means that underfitting and misspecification are not very likely to happen. Thus, the AIC can be used to select the correct type of TAR models with consistent estimate of threshold, but it is not likely to find the true threshold and order by the criterion of the AIC.

For a small or median difference of the persistence between the two regimes, the GAR models have the smallest SBC, the EAR, AR and ARMA models have relatively low SBC too and all the threshold models have larger SBC than other models. Thus, underfitting and misspecification are very likely to happen in this case if we use the SBC to select models.

For a large difference of persistence between the two regimes, either the true TAR or TAR-C models have the smallest SBC among all the models. Thus, underfitting and misspecification are not likely to happen in this case if we use the SBC to select models.

As to the MSPE for the out-of-sample forecast, for a large enough difference of the persistence between the two regimes, the true TAR models forecast the best among all the models, but for a small difference of the persistence between the two regimes, the AR models have the smallest MSPE. The distributions of the AIC, ABC and MSPE indicate the same results as above. Thus, the MSPE can select the correct TAR processes only if the difference of the persistence between the two regimes is large enough. Otherwise, the AR models forecast the best. However, if we don't know the true order and threshold of the TAR process, the MSPE can't select the TAR process because either the AR or EAR models have smaller MSPE than the TAR models in most cases. The TAR-C models forecast the best in only one case.

3.4 Summary

To sum up, for AR processes, if the AIC and SBC criteria are used to select models, the possibility of overfitting is very high since the non-linear models and other linear models are very likely to have lower AIC and SBC. However, the MSPE for one-step ahead out-of-sample forecast can be used to identify the true AR processes.

For TAR processes, if the AIC and SBC are used to select models, the possibility of underfitting and misspecification is very high for a small or median difference of the persistence between the two regimes; the possibility of underfitting and misspecification is very small for a large difference of the persistence between the two regimes. Also, it is not likely to find the true threshold and order. However, the AIC can be used to identify the TAR-C models. The SBC and MSPE can identify the true TAR process only if the difference of the persistence between the two regimes is large enough.

However, if we don't know the true AR or TAR process, the MSPE can't select the AR or TAR models in most cases. Thus, none of the AIC, SBC and MSPE can select the AR model for a given AR process with unknown order. For a TAR process, the AIC can consistently identify the TAR-C process and the SBC can identify the TAR-C process only if the difference of the persistency is large enough.

Notes

[1] The mean squared prediction error (MSPE) criterion measures the mean of the squared forecast errors. This is a very popular loss function to assess the forecast appropriateness of a model. $MSPE = \frac{\sum_{t=1}^{n} (y_t - \hat{y}_t)^2}{n}$, where $\hat{y}_t$ is the forecast value of y_t and n is the number of forecasts.

CHAPTER 4 ESTIMATION AND FORECAST OF THE TERM STRUCTURE OF INTEREST RATES IN U.S.

4.1 Literature Review

Many authors have been investigating the asymmetric adjustments of interest rates. Anderson (1994) estimated a smooth transition error-correction model of the U.S. treasury bill market. Balke and Fomby (1997) showed that various short-term interest rates exhibit threshold cointegration. Enders and Granger (1998) developed TAR and M-TAR unit-root tests[1], applied them to the term structure of interest rates and found that the term structure of interest rates is stationary with asymmetric adjustments toward the long-run equilibrium. They then used an asymmetric error-correction model with M-TAR adjustments to estimate the asymmetric adjustments of long-term and short-term interest rates to the discrepancy between the short-term and long-term rates. Enders and Siklos (1999) provided the empirical evidence of error-correction with M-TAR adjustment among interest rates of different maturities in U.S.. Guirauis (1994) discussed that the *ex ante* real and nominal interest rates respond asymmetrically to money innovations. The cautious policy or possibly opportunistic behavior of the Fed implies the asymmetric behavior of the term structure of interest rates. When the long-term interest rate (representing inflationary expectations) increases, the Fed adjusts the federal funds rate, an instrument of monetary policy, to decrease inflationary expectations. However, when the long-term interest rate decreases, the Fed does nothing.

However, none has been studies about how to estimate and forecast the term structure of interest rates. Toward this end, I will apply various non-linear time series models to the term structure of interest rates to study their in-sample estimating and out-of-sample forecasting performances.

4.2 Data Description

The empirical analysis uses the natural logarithm of the monthly yields of the federal funds rate (r_S) and 10-year interest rate on U.S. government securities (r_L) obtained from the CD-ROM version of the *International Financial Statistics* over the 1979:10 - 1997:04 period with a total of 211 observations (see Figure 4.1). The federal funds rate is chosen since it is an instrument of monetary policy and the 10-year yield is chosen since it represents an indicator of inflationary expectations. I begin the sample in 1979:10 because of changes in Federal Reserve operating procedures. It is generally accepted that short-term and long-term interest rates are I(1) and cointegrated such that the interest rate differential is stationary (see Stock and Watson, 1988). Let's define y as the spread of r_S and r_L and dy as the first-order difference of y, i.e., y = r_L - r_S and $dy_t = y_t - y_{t-1}$. The time path of y appears in Figure 4.2 and the plot of dy appears in Figure 4.3. From the plots, it seems that r_S and r_L move together except over the period of 1991-1994, when r_S and r_L drift away from each other.

The results of unit-root tests for y are reported in Table 4.1. Here, the Dickey-Fuller unit-root test statistics τ_μ, with value of -2.53, cannot reject the unit-root null hypothesis at the 5% significance level with one lag. The TAR unit-root test statistic, φ_μ, for y, with value

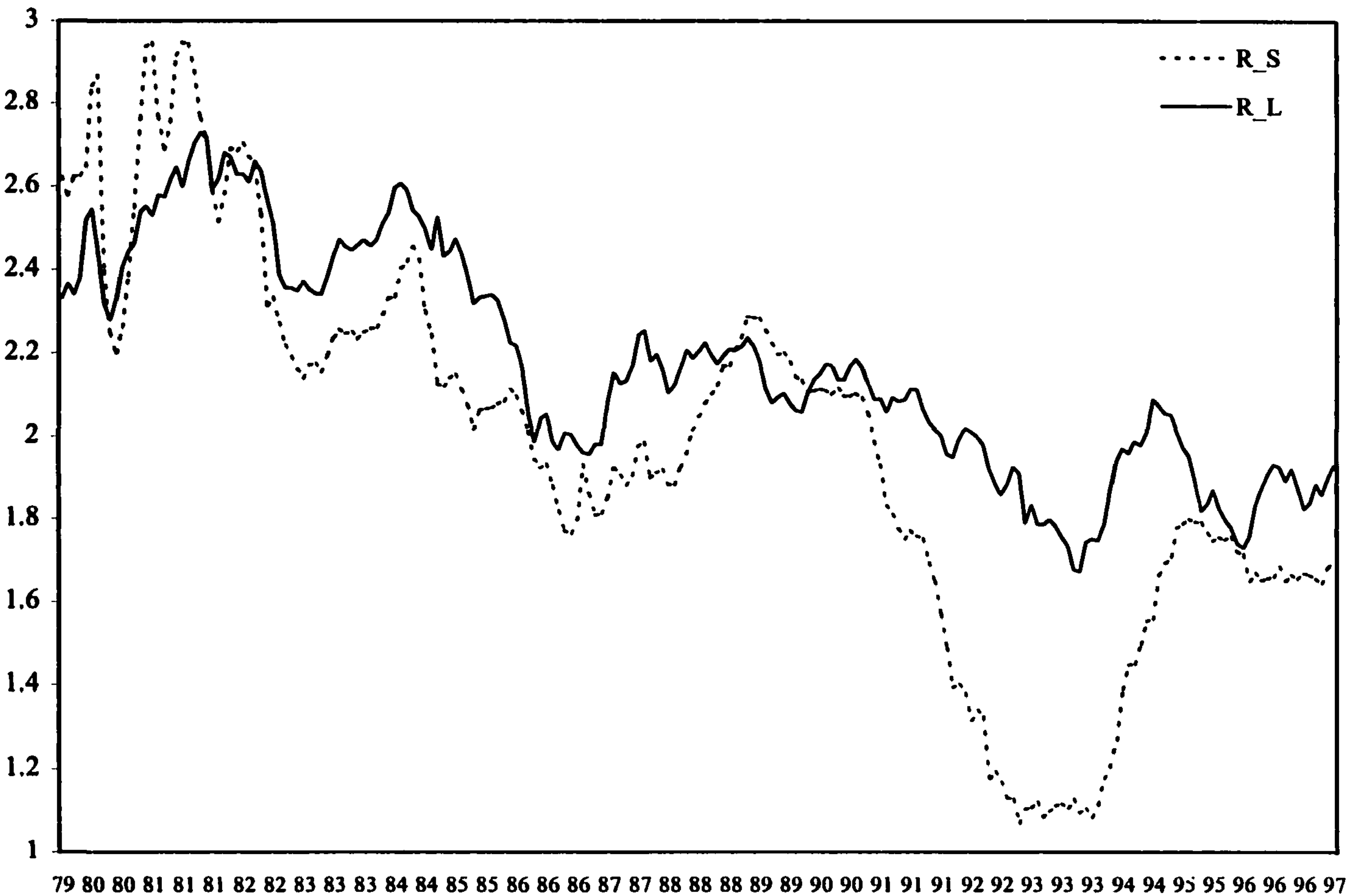


Figure 4.1 Natural Logrithm of the U.S. Interest Rates: R_S and R_L (79:10 - 97:04)

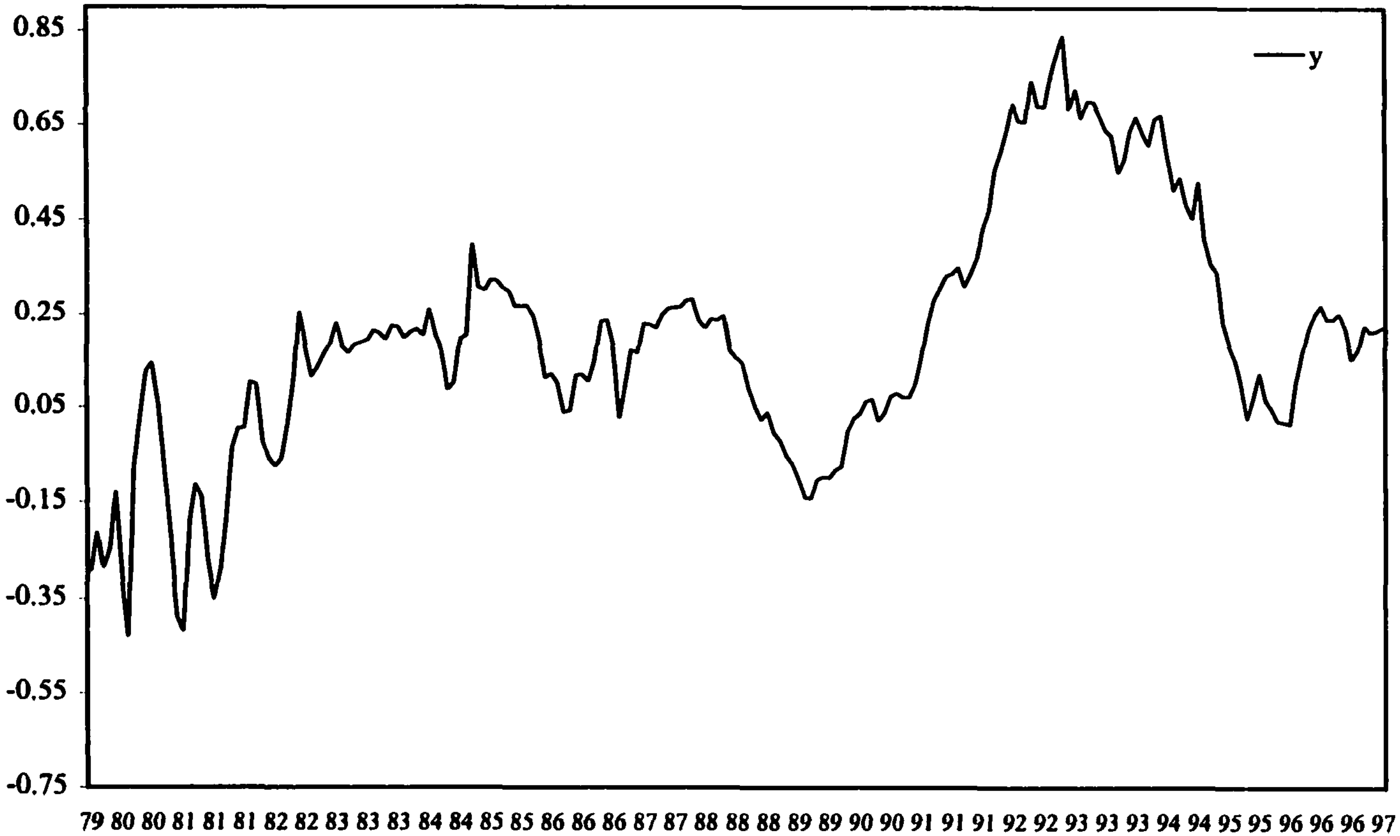


Figure 4.2 The Spread of U.S. Interest Rates, y (79:10 - 97:04)

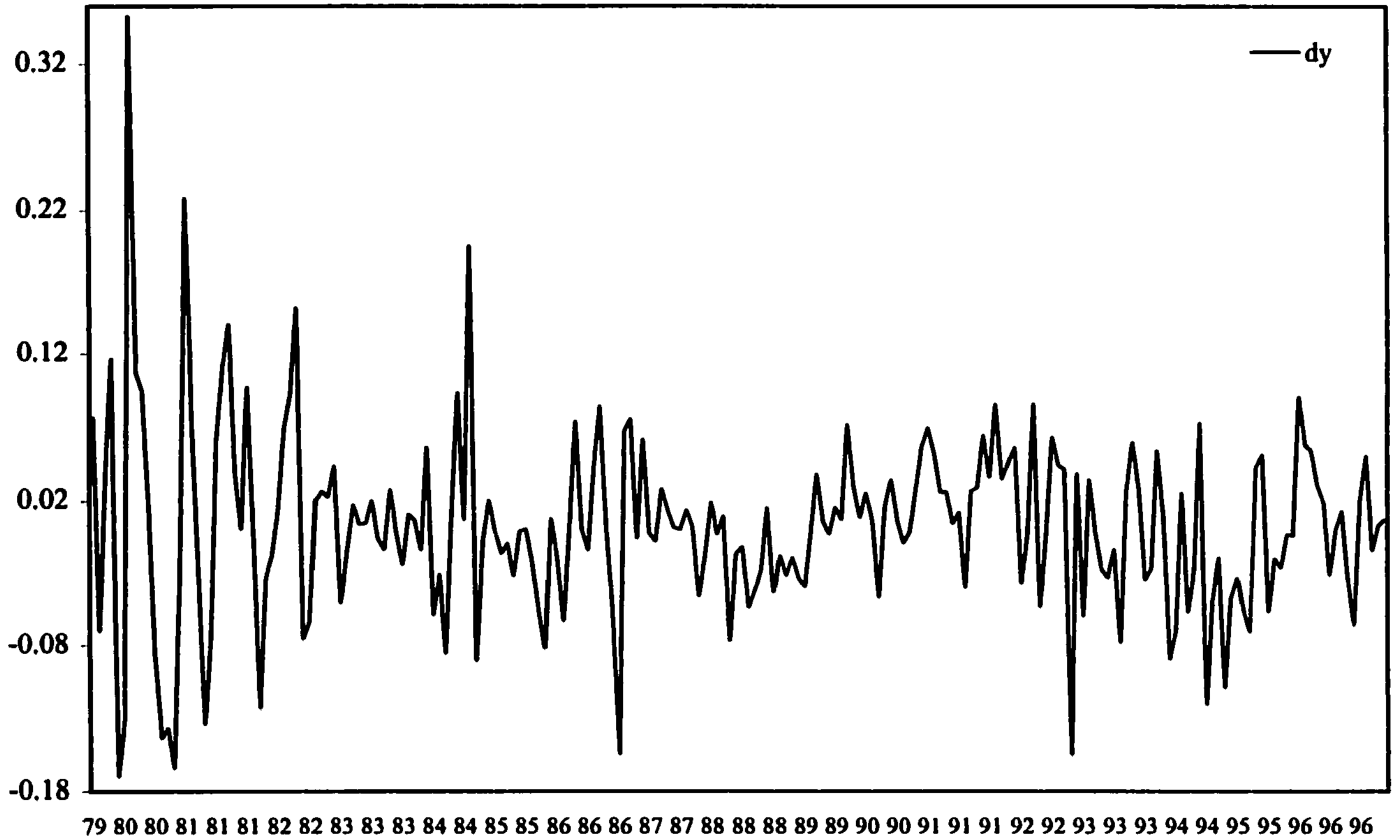


Figure 4.3 First-order Difference of the Spread of U.S. Interest Rates, dy (79:10 - 97:04)

Table 4.1 The Results for the Unit Root Tests of the Spread of the Natural Logarithm of Interest Rates in U.S. (79:10-97:10) [a]

Type of Test	Statistic	Value
Dickey-Fuller Unit-root Test	τ_{μ}	-2.53 (-2.88)[b]
TAR Unit-root Test [c]	φ_{μ}	5.18 (4.56)
	$H_0: \rho_1 = \rho_2$	3.89 (0.05)[d]
TAR Unit-root Test [c] with Consistent Threshold [e]	φ_{μ}	12.89 (4.56)
	$H_0: \rho_1 = \rho_2$	18.20 (0.00)
M-TAR Unit-root Test [c]	φ_{μ}^{*}	4.78 (4.95)
	$H_0: \rho_1 = \rho_2$	2.51 (0.11)
M-TAR Unit-root Test [c] with Consistent Threshold [e]	φ_{μ}^{*}	5.72 (4.95)
	$H_0: \rho_1 = \rho_2$	5.80 (0.02)

[a] The M-TAR with consistent threshold unit-root test has 2 lags, selected by SBC. The others have 1 lag.

[b] The numbers in the parentheses under the unit-root test statistics are the critical values at the 5% significance level.

[c] These are unit-root tests developed by Enders and Granger (1998).

[d] All numbers in the parentheses under the F-statistics for symmetry are the p-values.

[e] The consistent thresholds are estimated by Chan's (1993) method.

of 5.18, can reject unit-root at the 5% significance level, and the value of F statistic can reject the null hypothesis of symmetric adjustment with one lag. Then, the M-TAR unit-root test statistic, φ_{μ}^{*} , and the F statistic, with values of 4.78 and 2.51, are not large enough to reject the null hypotheses of unit-root and symmetric adjustment at the 5% significance level with one lag. In the end, Chan's (1993) method is used to get the consistent estimate of thresholds for TAR and M-TAR models and then the unit-root and symmetry tests are conducted. The TAR unit-root test statistic, φ_{μ}, and F statistic, with value of 12.89 and 18.20, reject the null hypotheses of unit-root and the symmetric adjustment at the 5% significance level with one lag. The M-TAR unit-root test statistic, φ_{μ}^{*}, and the F statistic, with the values of 5.72 and 5.80 respectively, are large enough to reject the null hypotheses of unit-root and symmetric adjustment at the 5% significance level with two lags.

Since the above results of the unit-root tests of y are kind of ambiguous, I estimate y and its first-order difference, dy, at the same time over the period of 1980:2 - 1997:4 (I started from the same period such that the AIC and SBC are directly comparable).

4.3 In-Sample Estimation Results

4.3.1 In-Sample Estimation Results of y

Table 4.2 reports the selected models for the spread of the natural logarithm of interest rates. The ARMA(1, 1) model is the best model among the ARMA models. The estimated residual autocorrelation function and the Ljung-Box Q statistic suggest that the model is adequate in the sense that there is little linear structure in the residual. However, from the results of the last column of Table 4.2, the non-linear time series models can reduce the

Table 4.2 Estimated Models for y, the Spread of the Natural Logarithm of Interest Rates in U.S. (80:2-97:10)

MODEL TYPE		ESTIMATED MODEL	AIC	SBC	RATIO[a]
ARMA(1,1)		$y_t = 0.246 + 0.940\ y_{t-1} + 0.288\ \hat{e}_{t-1}$ (2.64)[b] (41.63) (4.16)	-44.14	-34.14	
TAR	(1)[c]	$y_t = 0.990 y_{t-1}$, if $y_{t-1} \geq 0$ (75.84) $y_t = -0.049 + 1.165\ y_{t-1} - 0.988\ y_{t-2}$ (-2.90) (9.23) (-4.88) $+ 0.718 y_{t-3} - 0.289 y_{t-4}$, if $y_{t-1} < 0$ (3.03) (-1.98)	-62.68	-42.68	0.887
	(2)[d]	$y_t = 1.230\ y_{t-1}\ I_t + 1.024\ y_{t-1}\ (1 - I_t)$ (18.28) (12.70) $- 0.243\ y_{t-2}$ (-3.64) where $I_t = 1$ if $y_{t-1} \geq 0$, 0 if $y_{t-1} < 0$	-50.10	-40.10	0.972

[a] Ratio is the ratio of the residual variances of the nonlinear model and that of the ARMA(1, 1) model.

[b] All numbers in parentheses are T-statistics.

[c] TAR(1) and TAR-C(1) are TAR models with the form of (2.6) and (2.8).

[d] TAR (2) and TAR-C(2) are TAR models of the form of (2.5) and (2.6).

[e] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan, 1993).

[f] M-TAR(1) and M-TAR-C(1) are M-TAR models with the form of (2.7) and (2.8).

[g] M-TAR (2) and M-TAR-C(2) are M-TAR models of the form of (2.5) and (2.7).

Table 4.2 (continued)

MODEL TYPE		ESTIMATED MODEL	AIC	SBC	RATIO[a]
TAR-C[e]	(1)[c]	$y_t = 1.189\ y_{t-1} - 0.202\ y_{t-2}$, if $y_{t-1} \geq -0.083$ (16.73) (-2.86) $y_t = -0.134 + 1.100\ y_{t-1} - 1.294\ y_{t-2}$ (-4.46) (6.22) (-5.05) $+ 0.840\ y_{t-3} - 0.390\ y_{t-4}$, (3.23) (-2.52) if $y_{t-1} < -0.083$	-80.13	-53.47	0.798
	(2)[d]	$y_t = 1.225\ y_{t-1} I_t + 0.971\ y_{t-1}(1 - I_t)$ (18.32) (12.65) $- 0.237\ y_{t-2}$ (-3.60) where $I_t = 1$ if $y_{t-1} \geq -0.10186$, 0 if $y_{t-1} < -0.10186$	-53.25	-39.92	0.948
M-TAR	(1)[f]	$y_t = 0.032 + 0.918\ y_{t-1}$, if $dy_{t-1} \geq 0$ (3.92) (37.54) $y_t = 1.279\ y_{t-1} - 0.635\ y_{t-2} + 0.356\ y_{t-3}$ (11.10) (-3.81) (3.47) if $dy_{t-1} < 0$	-53.05	-36.39	0.939
	(2)[g]	$y_t = 1.252\ y_{t-1} I_t + 1.205\ y_{t-1}(1 - I_t)$ (17.56) (17.36) $- 0.446\ y_{t-2} + 0.195\ y_{t-3}$ (-4.06) (2.67) where $I_t = 1$ if $dy_{t-1} \geq 0$,0 if $dy_{t-1} < 0$	-43.30	-29.97	0.987

Table 4.2 (continued)

MODEL TYPE		ESTIMATED MODEL	AIC	SBC	RATIO[a]
M-TAR-C[e]	(1)[f]	$y_t = 0.038 + 0.918\ y_{t-1}$, if $dy_{t-1} \geq 0.0309$ (3.49) (32.43) $y_t = 1.267\ y_{t-1} - 0.580\ y_{t-2} + 0.306\ y_{t-3}$, (11.99) (-3.88) (3.38) if $dy_{t-1} < 0.0309$	-48.42	-28.42	0.950
	(2)[g]	$y_t = 1.255\ y_{t-1}\ I_t + 1.130\ y_{t-1}(1 - I_t)$ (18.28) (15.26) $- 0.434\ y_{t-2} + 0.174\ y_{t-3}$ (-4.14) (2.57) where $I_t = 1$ if $dy_{t-1} \geq 0.07249$, 0 if $dy_{t-1} < 0.07249$	-47.63	-30.97	0.961
GAR		$y_t = 1.242\ y_{t-1} - 0.338\ y_{t-2}$ (18.58) (-5.09) $- 0.611\ y_{t-1}^3 + 0.540\ y_{t-1}\ y_{t-2}$ (-5.04) (6.03)	-69.53	-56.20	0.874
EAR		$y_t = 1.336 \exp(-y_{t-1}^2)\ y_{t-1}$ (17.73) $+ [0.850 - 1.378 \exp(-y_{t-1}^2)]\ y_{t-2}$ (8.57) (-13.28) $+ 0.165\ y_{t-3}$ (2.44)	-51.08	-37.75	0.949
BILINEAR (2,0;1,2)		$y_t = 1.461\ y_{t-1} - 0.465\ y_{t-2} - 1.864\ y_{t-1}\hat{e}_{t-1}$ (23.75) (-7.65) (-12.38) $+ 0.959\ y_{t-1}\ \hat{e}_{t-2}$ (7.99)	-65.14	-48.44	0.884

residual variance[2].

Two forms of TAR models are estimated and both of them have much smaller AIC, SBC and residual variance than the ARMA(1,1) model. The TAR(1) model is the two-regime TAR model of the form of (2.6) and (2.8). The TAR(2) model is the two- regime TAR model of the form of (2.5) and (2.6), which indicates that the AR(2) process for the positive spread is a more persistent process than that for the negative. This means that negative shocks to the spread decay more quickly than to positive shocks. Chan's (1993) method is used to estimate the consistent thresholds. The TAR(1) model of the form of (2.6) and (2.8) with consistent estimate of the threshold is the best model of all the estimated models insofar as the AIC and residual variance are the smallest among all the models. The TAR(2) model with consistent estimate of the threshold has smaller AIC and residual variance, but larger SBC than the TAR(2) model.

Two forms of M-TAR models are estimated. The M-TAR(1) model is the two- regime TAR model of the form of (2.7) and (2.8), which is better than the ARMA(1,1) model. The M-TAR(2) model is the two-regime M-TAR model of the form of (2.5) and (2.7), which is worse than the ARMA(1,1) model. The M-TAR model exhibits larger decay for negative changes in the spread than for positive ones; i.e., increases are more persistent than decreases. Chan's (1993) method is used to estimate the consistent thresholds. The M-TAR(1) model with consistent estimate of the threshold is worse than that without consistent estimate of the threshold, but the M-TAR(2) model with consistent estimate of the threshold is better than that without consistent estimate of the threshold.

The best-fitting GAR model fits the data substantially better than the ARMA(1,1) model insofar as the AIC, SBC and variance of the residual are much smaller by adding the higher order components to the conventional AR model.

The best-fitting EAR model is the model with exponential coefficients of the first and second lags and it is better than the ARMA(1,1) model. The periods surrounding the turning points of a time series have similar dynamic structures, whereas the middle ground can have different dynamics. The coefficients of y_{t-1} and y_{t-2} depend on y_{t-1}. The larger the absolute values of the lagged spread deviations from its long-run equilibrium, the more persistent the dynamics of the EAR process insofar as the summation of the coefficients of y_{t-1} and y_{t-2} becomes larger.

The subset bilinear model is better than the ARMA model. This suggests that responses of the spread depend on its past observations and the correlation of the shocks and its past observations.

4.3.2 In-Sample Estimation Results of dy

Table 4.3 reports the selected models for the first-order difference of the spread of the natural logarithm of interest rates. The AR(4) model is the best model among the ARMA models. The estimated residual autocorrelation function and the Ljung-Box Q statistic suggest that there is no significant serial correlation in the residual. However, from the results of the last column of Table 4.3, the non-linear time series models may reduce the residual variance a little bit.

Two forms of TAR models are estimated. The TAR (1) model is the two- regime TAR

Table 4.3: Estimated Model for dy, the First-order Difference of the Spread of the Natural Logarithm of Interest Rates in U.S. (80:2-97:4)

MODEL		ESTIMATED MODEL	AIC	SBC	RATIO[a]
AR(4)		$dy_t = 0.228\ dy_{t-1} - 0.182\ dy_{t-2} - 0.165 dy_{t-4}$ (3.36)[b] (-2.67) (-2.48)	-46.18	-36.18	
TAR	(1)[c]	$dy_t = 0.196 dy_{t-1} - 0.184\ dy_{t-4}$, if $dy_{t-1} \geq 0$ (2.25) (-2.22) $dy_t = 0.279\ dy_{t-1} - 0.357\ dy_{t-2}$, if $dy_{t-1} < 0$ (2.71) (-3.52)	-49.18	-35.85	0.977
	(2)[d]	$dy_t = 0.231\ dy_{t-1}\ I_t + 0.238\ dy_{t-1}\ (1 - I_t)$ (2.56) (2.28) $- 0.165\ dy_{t-2}$ (-2.40) where $I_t = 1$ if $dy_{t-1} \geq 0$, 0 if $dy_{t-1} < 0$	-40.02	-30.03	1.031

[a] Ratio is the ratio of the residual variances of the nonlinear model and that of the AR(4) model.

[b] All numbers in parentheses are T-statistics.

c TAR(1) and TAR-C(1) are TAR models with the form of (2.6) and (2.8).

[d] TAR (2) and TAR-C(2) are TAR models of the form of (2.5) and (2.6).

[e] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan, 1993).

[f] M-TAR(1) and M-TAR-C(1) are M-TAR models with the form of (2.7) and (2.8).

[g] M-TAR (2) and M-TAR-C(2) are M-TAR models of the form of (2.5) and (2.7).

Table 4.3 (continued)

MODEL		ESTIMATED MODEL	AIC	SBC	RATIO[a]
TAR-C[e]	(1)[c]	$dy_t = 0.016 - 0.234\, dy_{t-4},$ if $dy_{t-1} \geq 0.007$ (2.51) (-2.77) $dy_t = 0.280\, dy_{t-1} - 0.358\, dy_{t-2},$ (2.75) (-3.67) if $dy_{t-1} < 0.007$	-50.98	-34.32	0.960
	(2)[d]	$dy_t = 0.197\, dy_{t-1} I_t + 0.265\, dy_{t-1}(1 - I_t)$ (2.16) (2.65) $- 0.182\, dy_{t-2} - 0.165\, dy_{t-4}$ (-2.67) (-2.47) where $I_t = 1$ if $dy_{t-1} \geq 0.03859$, 0 if $dy_{t-1} < 0.03859$	-42.44	-25.77	0.997
M-TAR	(1)[f]	$dy_t = -0.316 dy_{t-2} - 0.255\, dy_{t-4},$ if $d^2y_{t-1} \geq 0$ (-3.19) (-2.47) $dy_t = 0.345\, dy_{t-1},$ if $d^2y_{t-1} < 0$ (3.35)	-50.63	40.63	0.978
	(2)[g]	$dy_t = 0.130\, dy_{t-1} I_t + 0.386\, dy_{t-1}(1 - I_t)$ (1.48) (3.65) $- 0.170\, dy_{t-2}$ (-2.50) where $I_t = 1$ if $d^2y_{t-1} \geq 0$, 0 if $d^2y_{t-1} < 0$	-43.54	-33.55	1.007

Table 4.3 (continued)

MODEL		ESTIMATED MODEL	AIC	SBC	RATIO[a]
M-TAR-C[e]	(1)[f]	$dy_t = -0.577\, dy_{t-2} - 0.317\, dy_{t-4,}$ (-5.37) (-2.70) if $d^2y_{t-1} \geq 0.03$ $dy_t = 0.386\, dy_{t-1}$, if $d^2y_{t-1} < 0.03$ (4.49)	-73.17	-59.84	0.871
	(2)[g]	$dy_t = -\, 0.007\, dy_{t-1} I_t + 0.452\, dy_{t-1}(1 - I_t)$ (-0.07) (4.87) $-\, 0.228 dy_{t-2} - 0.145 dy_{t-4}$ (-3.37) (-2.23) where $I_t = 1$ if $d^2y_{t-1} \geq 0.03774$, 0 if $d^2y_{t-1} < 0.03774$	-53.88	-37.21	0.997
GAR		$dy_t = 0.228\, dy_{t-1} - 0.238\, dy_{t-2}$ (3.41) (-3.41) $-\, 0.141\, dy_{t-4} + 1.098\, dy_{t-2}^2$ (-2.14) (2.84)	-52.23	-38.90	0.962
EAR		$dy_t = 0.228\, dy_{t-1} - 0.181 dy_{t-2}$ (3.35) (-2.67) $-\, 0.166 \exp(-\, dy_{t-1}^2) dy_{t-4}$ (-2.49)	-46.20	-36.21	1.000
BILINEAR (0,1;2,2)		$dy_t = 0.261\, \hat{e}_{t-1} - 1.611\, dy_{t-1}\, \hat{e}_{t-1}$ (4.24) (-4.86) $+\, 6.898\, dy_{t-1} \hat{e}_{t-2} - 5.308\, dy_{t-2}\, \hat{e}_{t-1}$ (7.71) (-7.00)	62.96	46.30	0.906

model of the form of (2.6) and (2.8), which has smaller AIC and residual variance, but larger SBC than the AR(4) model. The TAR (2) model is the two- regime TAR model of the form of (2.5) and (2.6), which is worse than the AR(4) model. From the TAR(2) model, the AR(2) model for the negative changes in the spread is a more persistent process than that for the positive ones. Thus, this estimated model is consistent with the asymmetric federal reserve policy. An increases in the long-term rate (representing an increase in inflationary expectations) is met with policy adjustments to decrease inflationary expectations. But the decreases in the long-term rate are allowed to persistent. Chan's (1993) method is used to estimate the consistent thresholds. The explanation is the same as the TAR model.

Two forms of M-TAR models are estimated. The M-TAR (1) model is the two-regime TAR model of the form of (2.7) and (2.8), which is better than the AR(4) model. The M-TAR (2) model is the two-regime M-TAR model of the form of (2.5) and (2.7), which is worse than the ARMA (1,1) model. The M-TAR model exhibits little decay for the decelerating deviations from its long-run equilibrium but substantial decay for accelerating ones; i.e., decelerating deviations from its long-run equilibrium are more persistent than accelerating ones. But since the t-statistic of the coefficient for dy_{t-1} when $d^2y_{t-1} \geq 0$ is not significant, the spread responds significantly to decelerating deviations from its long-run equilibrium, but does not respond significantly to accelerating ones. At the same time, Chan's (1993) method is used to estimate the consistent thresholds. Both of the M-TAR models with consistent estimate of the thresholds are much better than the corresponding M-TAR models without consistent estimate of the thresholds and the AR(4) model. In fact, the

M-TAR-C model of the form of (2.7) and (2.8) is the best among all the models.

The best-fitting GAR model can be writen as:

$dy_t = 0.228\ dy_{t-1} - (0.238 - 1.098\ dy_{t-2})\ dy_{t-2} - 0.141 dy_{t-4}$.

Thus, the coefficient of dy_{t-2} is a smooth function of dy_{t-2}, and the larger the values of the past change in the spread the more persistent the estimated GAR model. This model fits the data a little better than the AR(4) models in the sense that the AIC, SBC and residual variance of this model are smaller .

The best-fitting EAR model is the one with an exponential coefficient of the fourth lag, which has almost the same AIC, SBC and residual variance as the AR(4) model. The larger absolute values of the lagged change in the spread, the more persistent the dynamics of the EAR process. This is consistent with the results of the GAR model.

The subset bilinear model is reported in the last row of Table 4.3. This model is substantially better than the AR(4) model insofar as the AIC, SBC and residual variance are much smaller. This suggests that responses of the changes of the spread depend on the shocks and the correlation of shocks and past changes in the spread.

4.4 Out-of-Sample Forecasting Performance Results

4.4.1 Method and Statistics Overview

The forecast of the spread of interest rates is applied over the period of 1988:2-1997:4 with a total of 111 forecast values. The 1-step ahead forecast of each model is estimated recursively, i.e., the data through period t is used to make the 1-step ahead forecast in period t+1 for each model. The bias[3] and its t-statistic are calculated by regressing the forecast error on a constant for each model. MSPE ratios are the ratios of the MSPE of each non-linear model to that of the ARMA model. Mizrach's (1995) robust forecast comparison statistic is used to test the null hypothesis that MSPE ratio equals 1.

Mizrach's (1995) robust forecast comparison statistic is used to test the null hypothesis that MSPEs are equal for two models, i.e., MSPE ratio for two models equals 1. The idea and statistic are described as follows:

Let $e_{1,t}$ and $e_{2,t}$ be the prediction errors of period t from model 1 and 2, respectively, where t = 1, ..., n. Mizrach's (1995) robust forecast comparison test assumes that the two prediction errors are stationary processes from a bivariate population (E_1, E_2). Let $U = E_1 - E_2$ and $V = E_1 + E_2$. If the MSPEs in the original population are equal, the covariance in the transformed series is zero. Thus, the null hypothesis of Mizrach's (1995) robust forecast comparison test is cov(U,V) = 0. Let $u_t = e_{1,t} - e_{2,t}$ and $v_t = e_{1,t} + e_{2,t}$. Then the robust forecast comparison test statistic is:

$$COVSTAT = \frac{(1/n)\sum_{t=1}^{n} u_t v_t}{\left\{ \sum_{j=-k}^{k} \left[1 - |j|/(k+1)\right] s_{uvuv}(j) \right\}^{1/2}} \tag{4.1}$$

where k is the step of the forecasts and $s_{uvuv}(j) = \frac{1}{n}\sum_{t=j+1}^{n} u_t v_t u_{t-j} v_{t-j}$. The asymptotic distribution of COVSTAT is standard normal.

4.4.2 Out-of-Sample Forecasting Performance Results of y

The biases and their t-statistics, the MSPE ratios and the p-values of the Mizrach's robust forecast comparison statistics (they are in parentheses under MSPE ratios) are reported in Table 4.4. There is no evidence of significant forecast bias for all models since all the t-statistics are very small.

The GAR model forecasts the best of all the models. The TAR(2), M-TAR(2), M-TAR-C and bilinear models generate lower MSPEs than the ARMA(1,1) model, but all the MSPE reductions are not statistically significant since all the p-values of the Mizrach's robust forecast comparison statistics are very large. The other models generate higher MSPEs than the ARMA(1, 1) model even though these increases in MSPE are not statistically significant either. Thus, non-linear time series models can forecast out-of-sample better than the ARMA model, even though they do not significantly dominate the ARMA model.

4.4.3 Out-of-Sample Forecasting Performance Results of dy

The same rules as those for the spread the natural logarithm of interest rates are applied to forecast and assess the first-order difference of the spread the natural logarithm of interest rates over the same period of 1988:2-1997:4. The results are reported in Table 4.5. There is no evidence of significant forecast bias for all models since all the t-statistics are very small.

The TAR model of the form of (2.5) and (2.6) forecasts the best of all the models. The TAR and TAR-C(1) models, the M-TAR(2) model and the EAR model generate lower MSPEs than the AR(4) model, but all of the MSPE reductions are not statistically significant

Table 4.4 Bias and MSPE Ratios of Different Models for y, the Spread of the Natural Logarithm of Interest Rates in U.S. (88:2-97:4)

MODEL		BIAS [a]	MSPE RATIOS [b]
ARMA		-0.005 (-0.99)	
TAR	(1)[c]	-0.005 (-1.04)	0.965 (0.97)
	(2)[d]	-0.001 (-0.30)	0.882 (0.87)
TAR-C[e]	(1)[c]	-0.008 (-1.54)	1.216 (0.92)
	(2)[d]	0.002 (0.35)	0.855 (0.88)
M-TAR	(1)[f]	-0.001 (-0.23)	1.005 (0.99)
	(2)[g]	-0.005 (-1.24)	0.873 (0.83)
M-TAR-C[e]	(1)[f]	0.001 (0.21)	0.916 (0.85)
	(2)[g]	0.002 (0.36)	0.860 (0.88)
GAR		-0.001 (-0.26)	0.763 (0.81)
EAR		-0.007 (-1.41)	1.102 (0.91)
BILINEAR		-0.004 (-0.96)	0.856 (0.93)

[a] Bias is calculated by regressing the forecast error on a constant; the number below is its t-statistic.

[b] MSPE Ratios are the ratios of MSPEs of the nonlinear models to that of the ARMA model; the numbers in parentheses are the p-values for the Mizrach's (1995) robust forecast comparison statistic, testing that the MSPE ratio equals 1.

[c] TAR(1) and TAR-C(1) are TAR models with the form of (2.6) and (2.8).

[d] The TAR (2) and TAR-C(2) are TAR models of the form of (2.5) and (2.6).

[e] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan,1993).

[f] M-TAR(1) and M-TAR-C(1) are M-TAR models with the form of (2.7) and (2.8).

[g] The M-TAR (2) and M-TAR-C(2) are M-TAR models of the form of (2.5) and (2.7).

Table 4.5: Bias and MSPE Ratios of Different Models for dy, the First-order Difference of the Spread of the Natural Logarithm of Interest Rates in U.S. (88:2-97:4)

MODEL		BIAS[a]	MSPE RATIO[b]
AR(4)		0.000 (0.07)	
TAR	(1)[c]	-0.000 (-0.06)	0.947 (0.91)[b]
	(2)[d]	0.000 (0.06)	0.896 (0.78)
TAR-C[e]	(1)[c]	0.006 (1.26)	0.992 (0.99)
	(2)[d]	0.001 (0.27)	1.018 (0.93)
M-TAR	(1)[f]	0.001 (0.28)	1.075 (0.92)
	(2)[g]	-0.003 (-0.57)	0.933 (0.89)
M-TAR-C[e]	(1)	0.004 (0.81)	1.186 (0.89)
	(2)	-0.003 (-0.69)	1.066 (0.96)
GAR		0.003 (0.63)	1.067 (0.90)
EAR		0.000 (0.07)	0.978 (0.91)
BILINEAR		0.001 (0.15)	1.031 (0.98)

[a] Bias is calculated by regressing the forecast error on a constant; the number below is its t-statistic.

[b] MSPE Ratios are the ratios of MSPEs of the nonlinear models to that of the AR(4) model; the numbers in parentheses are the p-values for the Mizrach's (1995) robust forecast comparison statistic, testing that the MSPE ratio equals 1.

[c] TAR(1) and TAR-C(1) are TAR models with the form of (2.6) and (2.8).

[d] The TAR (2) and TAR-C(2) are TAR models of the form of (2.5) and (2.6).

[e] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan,1993).

[f] M-TAR(1) and M-TAR-C(1) are M-TAR models with the form of (2.7) and (2.8).

[g] The M-TAR (2) and M-TAR-C(2) are M-TAR models of the form of (2.5) and (2.7).

because all the p-values of the Mizrach's (1995) robust forecast comparison statistic are very large. The other models generate higher MSPEs than the AR(4) model even though these increases in MSPE are not statistically significant either. Thus, the AR(4) model is not statistically significantly dominated by non-linear models for the out-of-sample forecast. Since all the p-values are very high, there is no statistically significant difference in the MSPE between the non-linear time series models and the AR(4) model.

4.5 Monte Carlo Simulation Results

Even though the non-linear time series models can estimate and forecast better than the ARMA model, the MSPE reductions of the non-linear time series models are not statistically significant and the results of in-sample estimation and out-of-sample forecast are not always consistent. To check whether the dominance of non-linear time series models results from non-linearity of the term structure of interest rates or from overfitting, I apply the previous Monte Carlo simulation again by generating a true ARMA(1,1) process with the parameters are the same as the estimated parameters of y (The simulation of previous chapter is conducted on AR processes, not on ARMA processes). That is, I generate the following ARMA(1, 1) process:

$$y_t = 0.246 + 0.94y_{t-1} + 0.288e_{t-1} + e_t, t = 1, ..., 100$$

Then I estimate it as the true ARMA(1,1), ARMA, AR, bilinear, EAR, GAR, TAR-C and M-TAR-C models using the same searching rule as before. Finally, one-step ahead recursive forecast is used to forecast the period of t = 51, ..., 100. The means of the AIC, SBC and MSPE are reported in Table 4.6. The M-TAR-C model has the smallest AIC, GAR

Table 4.6 The Means of the AIC, SBC and MSPE for the True ARMA Model:
y(t) = 0.246 + 0.94*y(t-1)+0.288*e(t-1) + e(t)

ARMAT [a] _AIC	446.2126
ARMA_AIC	447.2431
BL_AIC	447.4484
AR_AIC	**446.1170** [c]
EAR_AIC	445.7011
GAR_AIC	445.3037
TARC_AIC	444.1361
MTARC_AIC	<u>443.6905</u> [b]
ARMAT_SBC	453.9368
ARMA_SBC	453.2628
BL_SBC	453.9186
AR_SBC	**451.7839**
EAR_SBC	451.5328
GAR_SBC	<u>450.8934</u>
TARC_SBC	457.4422
MTARC_SBC	455.8406
ARMAT_MSPE	<u>1.1010</u>
ARMA_MSPE	1.1448
BL_MSPE	1.1526
AR_MSPE	**1.1240**
EAR_MSPE	1.1314
GAR_MSPE	1.1751
TARC_MSPE	1.3329
MTARC_MSPE	1.2673

[a] ARMAT stands for the true ARMA(1,1) model with known order.

[b] The underlined numbers are the smallest among all the models for the AIC, SBC and MSPE.

[c] The bold numbers are the smallest AIC, SBC and MSPE among the true ARMA, ARMA and AR models.

model has the smallest SBC of all the models and the true ARMA model has the smallest MSPE. These results are the same as those of the AR processes. If we do not know the true order of the ARMA process, all the AIC, SBC and MSPE select the AR model for the linear model. If we consider the ARMA model, only the EAR model has a smaller MSPE than the ARMA model and all other non-linear time series models have larger MSPEs than the ARMA model. Thus, if the true process is an ARMA process, it is quite possible to select the AR model and the forecast of non-linear models can not beat the AR mode. At the same time, it is very unlikely that GAR, TAR-C, MTAR-C and bilinear models can generate lower MSPEs than the ARMA model with unknown order. Since the simulation results are not consistent with the results of y, I conclude that the dominance of non-linear time series models results from non-linearity of the term structure of interest rates, not from overfitting.

4.6 Conclusions

The results are summarized in Table 4.7. For the spread, the GAR model forecasts the best. The TAR, TAR-C of the form of (2.5) and (2.6), GAR and bilinear models are better than the ARMA model for both in-sample estimation and out-of-sample forecast. For the first-order difference of the spread, the TAR model of the form of (2.5) and (2.6) forecasts the best, even though it is worse than the AR model for in-sample estimation. The EAR model is better than the AR model for both the in-sample estimation and out-of-sample forecast. Also, it is not very likely that the dominance of non-linear models results from overfitting. Thus, non-linear models can dominate ARMA models for the in-sample estimation and out-of-sample forecast of the term structure of interest rates, even though

Table 4.7 The Summary of the Estimation and Forecast Results for the Spread and the First-order Difference of the Spread of Interest Rates in U.S.

Models		Spread		First-order Difference of the Spread	
		In-sample	Out-of-sample	In-sample	Out-of-sample
TAR	(1)[a]	B[b]	B	A[c]	B
TAR	(2)[d]	B	B	W[e]	**B**[f]
TAR-C[g]	(1)[a]	**B**	W	A	B
TAR-C[g]	(2)[d]	B	B	W	W
M-TAR	(1)[h]	B	W	B	W
M-TAR	(2)[i]	W	B	W	B
M-TAR-C[g]	(1)[h]	A	B	**B**	W
M-TAR-C[g]	(2)[i]	A	B	B	W
EAR		B	W	B	B
GAR		B	**B**	B	W
Bilinear		B	B	B	W

[a] TAR(1) and TAR-C(1) are TAR models with the form of (2.6) and (2.8).

[b] B means that the nonlinear model is better than the ARMA model in term of the AIC and SBC for in-sample estimation or the MSPE for out-of-sample forecast.

[c] A stands for ambiguity, i.e., the nonlinear model has a smaller AIC, but a larger SBC than the ARMA model.

[d] The TAR (2) and TAR-C(2) are TAR models of the form of (2.5) and (2.6).

[e] W means that the nonlinear model is worse than the ARMA model in term of the AIC and SBC for in-sample estimation or the MSPE for out-of-sample forecast.

[f] **B** means the best model among all the models.

[g] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan,1993).

[h] M-TAR(1) and M-TAR-C(1) are M-TAR models with the form of (2.7) and (2.8).

[i] The M-TAR (2) and M-TAR-C(2) are M-TAR models of the form of (2.5) and (2.7).

none of the non-linear models significantly dominates the ARMA model for the out-of-sample forecast.

Notes

[1] See Enders and Granger (1998) for detail.

[2] The residual variance, $\hat{\sigma}_e^2 = \frac{\sum_{t=1}^{T} (\hat{e}_t - \bar{\hat{e}})^2}{T-1}$, where $\hat{e}_t = y_t - \hat{y}_t$ is the estimated residual, $\bar{\hat{e}} = \frac{\sum_{t=1}^{T} \hat{e}_t}{T}$ is the mean of the estimated residuals and T is the number of usable observations.

[3] Bias measures the mean of forecast errors. $BIAS = \frac{\sum_{t=1}^{n} (y_t - \hat{y}_t)}{n}$, where $\hat{y}_t$ is the forecasted value of y_t and n is the number of forecasts.

CHAPTER 5 ESTIMATION AND FORECAST OF THE PRICE SPREAD IN THE U.S. PORK SECTOR

5.1 Literature Review

It is generally acknowledged that middlemen in the agricultural markets pass on input price increases to customers more rapidly and completely than input price decreases. Several theoretical and institutional reasons of asymmetric price response have been proposed. First, cost. Ward (1982) pointed out that agents may be hesitant to raise prices of perishable goods for fears of holding spoiled stocks. Bailey and Brorsen (1989) argued that asymmetric adjustment costs for price increases and decreases may be the reason of asymmetric price adjustments. Ball and Mankiw (1994) presented a menu-cost model with a positive trend inflation to show that prices respond more strongly to positive shocks than to negative shocks. Secondly, different market power for different levels of marketing chains. The agricultural markets are often less concentrated at the farm level than the wholesale and retail levels. For example, Oligopolistic processors may response collusively more rapidly to shocks that squeeze the margin than to shocks that stretch the margin. This asymmetric responses may result in asymmetric price transmission (Goodwin, 1994; Weaver et al., 1989; von Cramon-Taubadel,1998). Thirdly, government intervention (Kinucan and Forker, 1987). For example, if policies support producer prices, processors may reluctant to response to input price decreases, but adjust output prices quickly to input price increases because they expect that the decreases in producer prices will be more transient than increases in producer prices. Finally, Kinucan and Forker (1987) attributed asymmetric price transmission in dairy markets to the shocks originating from demand shifts by arguing that farm demand is less elastic than retail demand.

Lots of research has investigated the interrelationship and transmission of prices at farm, wholesale and retail markets for vegetable, meat and livestock products. In general, most of the research reached the following results: (1) existence of significant lag adjustments in prices at the various market levels (Boyd and Brorsen, 1988; Hahn, 1990); (2) the existence of asymmetric price adjustments at the various market levels though the extent of asymmetry is generally modest; (3) transmission of shocks flows from farm to wholesale to retail markets but not in the opposite direction. In particular, farm prices are relatively unresponsive to shocks in wholesale and retail markets and retail prices are responsive to shocks at farm level. For example, Ward (1982) found both short-run and long-run asymmetry in vertical price transmission of fresh vegetables in U.S., while Zhang et. al. (1995) found that the transmission from peanut to peanut butter prices in U.S. is asymmetric in the short-run, but symmetric in the long-run. Kinnucan and Forker (1987) showed the asymmetric farm-retail price transmission for New York State apples, and Hansmire and Willett (1992) reached the same conclusion for the dairy product in U.S.. Hahn (1990) showed that prices at all levels of the U.S. beef and pork market chains are more responsive to positive shocks than to negative shocks even though Boyd and Brorsen (1988) found no evidence of asymmetric vertical price transmission in the U.S. pork market.

Pick et al. (1990) found that the short-run, but not the long-run, vertical price transmission on the U.S. citrus market is asymmetric. Appel (1992) found that both the speed and magnitude of price transmission from the farmer to the retailer of broilers in Germany is asymmetric. Most of these studies confined to the standard model for estimating irreversibility developed by Wolffram (1971) and later modified by Houck (1977) and Ward (1982). That is, the response of a price P_i to another price P_j is estimated with equation (5.1):

$$\sum_{t=1}^{\tau} \Delta P_{i,t} = \alpha_0 + \alpha^+ \sum_{t=1}^{\tau} \Delta P_{j,t}^+ + \alpha^- \sum_{t=1}^{\tau} \Delta P_{j,t}^- + \varepsilon_t \qquad (5.1)$$

where ΔP^+ and ΔP^- are the positive and negative changes in P respectively, α_0, α^+ and α^- are coefficients and τ is the current time period. The symmetric test is the test of the null hypothesis: H_0: $\alpha^+ = \alpha^-$. Some authors estimated the equation (5.2) and then test the short-run and long-run asymmetry, respectively.

$$\sum_{t=1}^{\tau} \Delta P_{i,t} = \alpha_0 + \sum_{k=0}^{\lambda} \alpha_{\tau-k}^+ \left(\sum_{t=1}^{\tau-k} \Delta P_{j,t}^+ \right) + \sum_{k=0}^{\lambda} \alpha_{\tau-k}^- \left(\sum_{t=1}^{\tau} \Delta P_{j,t}^- \right) + \varepsilon_t \qquad (5.2)$$

where ΔP^+ and ΔP^- are the positive and negative changes in P respectively, α_0, $\alpha_{\tau-k}^+$ and $\alpha_{\tau-k}^-$ are coefficients, λ is the lag period and τ is the current time period. Then, the long-run symmetric test is the test of the null hypothesis: H_0: $\sum_{k=0}^{\lambda} \alpha_{\tau-k}^+ = \sum_{k=0}^{\lambda} \alpha_{\tau-k}^-$, and the short-run symmetry test is the test of the null hypothesis: H_0: $\alpha_{\tau-k}^+ = \alpha_{\tau-k}^-$, *for* $k = 0, 1, ..., \lambda$.

von Cramon-Taubadel (1998) showed that the estimation of Wolffram-Houck model does not adequately consider the time series properties of the data. That is, given the nonstationarity in individual prices, the equation (5.1) is a spurious regression and is inconsistent with cointegration. To see this, we can substitute (5.3) to (5.1) and get (5.4) by rearranging (Ward, 1982: 206).

$$\sum_{t=1}^{\tau} \Delta P_t = \sum_{t=1}^{\tau} \Delta P_t^+ + \sum_{t=1}^{\tau} \Delta P_t^- = P_\tau - P_0 \tag{5.3}$$

$$P_{i,\tau} = (P_{i,0} + \alpha_0 - \alpha^- P_{j,0}) + \alpha^- P_{j,\tau} + (\alpha^+ - \alpha^-) \sum_{t=1}^{\tau} \Delta P_{j,t}^+ + \varepsilon_\tau \tag{5.4}$$

If P_i and P_j are I(1), which is usually true in practice, then $\sum_{t=1}^{\tau} \Delta P_{j,t}^+$ is also I(1). So there are four cases related to (5.4) depending on $\alpha^+ - \alpha^-$ and ε_t:

I. $\alpha^+ - \alpha^- \neq 0$ (asymmetry) and ε_t is I(0). Then P_i, P_j and $\sum_{t=1}^{\tau} \Delta P_{j,t}^+$ are cointegrated and P_i and P_j are not cointegrated.

II. $\alpha^+ - \alpha^- \neq 0$ (asymmetry) and ε_t is I(1). Then because of non-stationary of ε_t, (5.1) is a spurious regression.

III. $\alpha^+ - \alpha^- = 0$ (symmetry) and ε_t is I(1). Then (5.1) is a spurious regression too.

IV. $\alpha^+ - \alpha^- = 0$ (symmetry) and ε_t is I(0). Then P_i and P_j are cointegrated.

Thus, Wolffram-Houck model is inconsistent with the asymmetry and cointegration. This problem can be solved by error-correction model if prices are cointegrated. Following

Granger and Lee's (1989) modification of an error-correction representation, von Cramon-Taubadel proposed an asymmetric error-correction representation of (5.5) and (5.6) to test for asymmetric price transmission:

$$P_{i,t} = \beta_0 + \beta_1 P_{j,t} + u_t \qquad (5.5)$$

$$\Delta P_{i,t} = \alpha_0 + \alpha_1^+ ECT_{t-1}^+ + \alpha_1^- ECT_{t-1}^- + \alpha_2 \Delta P_{j,t} + \alpha_3(L)\Delta P_{i,t-1} + \alpha_4(L)\Delta P_{j,t-1} + \varepsilon_t \qquad (5.6)$$

where (5.5) is the cointegrating relation, $\alpha_3(L)$ and $\alpha_4(L)$ are lag polynomials, the error-correction term is:

$$ECT_{t-1} = u_{t-1} = P_{i,t-1} - \beta_0 - \beta_1 P_{j,t-1} \qquad (5.7)$$

ECT_{t-1}^+ and ECT_{t-1}^- are the positive and negative ECT_{t-1}, respectively. Then the symmetry test is the test of the null hypothesis: H_0: $\alpha_1^+ = \alpha_1^-$.

von Cramon-Taubadel then applied the asymmetric error-correction model to the transmission of weekly producer and wholesale pork prices in northern Germany and found the asymmetric price transmission in the sense that the margin is corrected more rapidly when it is squeezed relative to its long-run level than when it is stretched. Goodwin and Holt (1999) used a three-regime threshold error-correction model to examine the price interrelationship and transmission among farm, wholesale and retail level in the U.S. beef marketing channel and found that prices respond asymmetrically to positive and negative shocks and shocks are largely unidirectional with information flowing from farm to wholesale to retail market, but not the opposite direction. Harper and Goodwin (1999) applied the same technique as Goodwin and Holt (1999) to the U.S. pork sector and got similar conclusions.

An interesting issue here is how to model the asymmetric adjustments and which model can forecast the best for the spread of prices at wholesale and retail markets for the U.S. pork sector. Towards this end, I will apply the previous non-linear time series models to the U.S. pork sector.

5.2 Data Description

The data used here are the monthly data of wholesale and farm prices of pork of U.S. from January 1970 to June 1999, with a total of 354 observations (see Figure 5.1). Both were collected from USDA Economic Research Service data, with units of cents per retail pound. From the construction, the two prices are comparable. To investigate the asymmetric property of the adjustment, I study the spread, the logarithm of the ratio of the wholesale and farm prices. That is, I define y = spread = log(wholesale/farm) (see Figure 5.2). Constructing the spread this way will avoid finding an appropriate price index to get real prices. Otherwise, different price index may lead to different results. Let's define dy is the first-order difference of y, i.e., $dy_t = y_t - y_{t-1}$ (see Figure 5.3). From the figure, the wholesale and farm prices move together. There is a big strike in the spread at the end of 1998. The farm prices decline significantly with a low of 29.3, which is well below the feed cost, in late 1998, but the magnitude decline of wholesale prices is not as large as that of the farm prices and thus the spread hits its historical high value in December 1998.

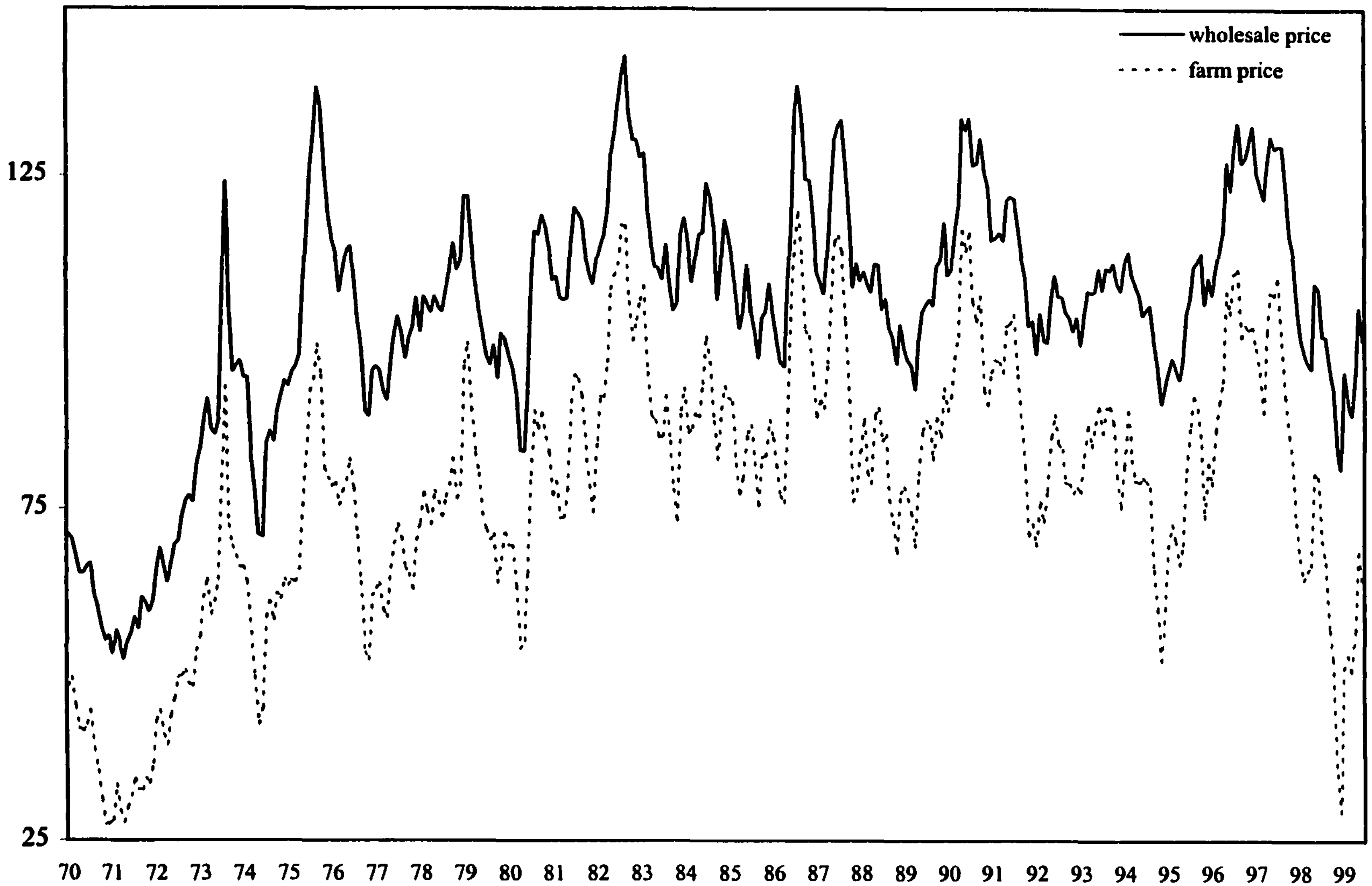


Figure 5.1 The Wholesale and Farm Prices of Pork in U.S. (70:1 - 99:6)

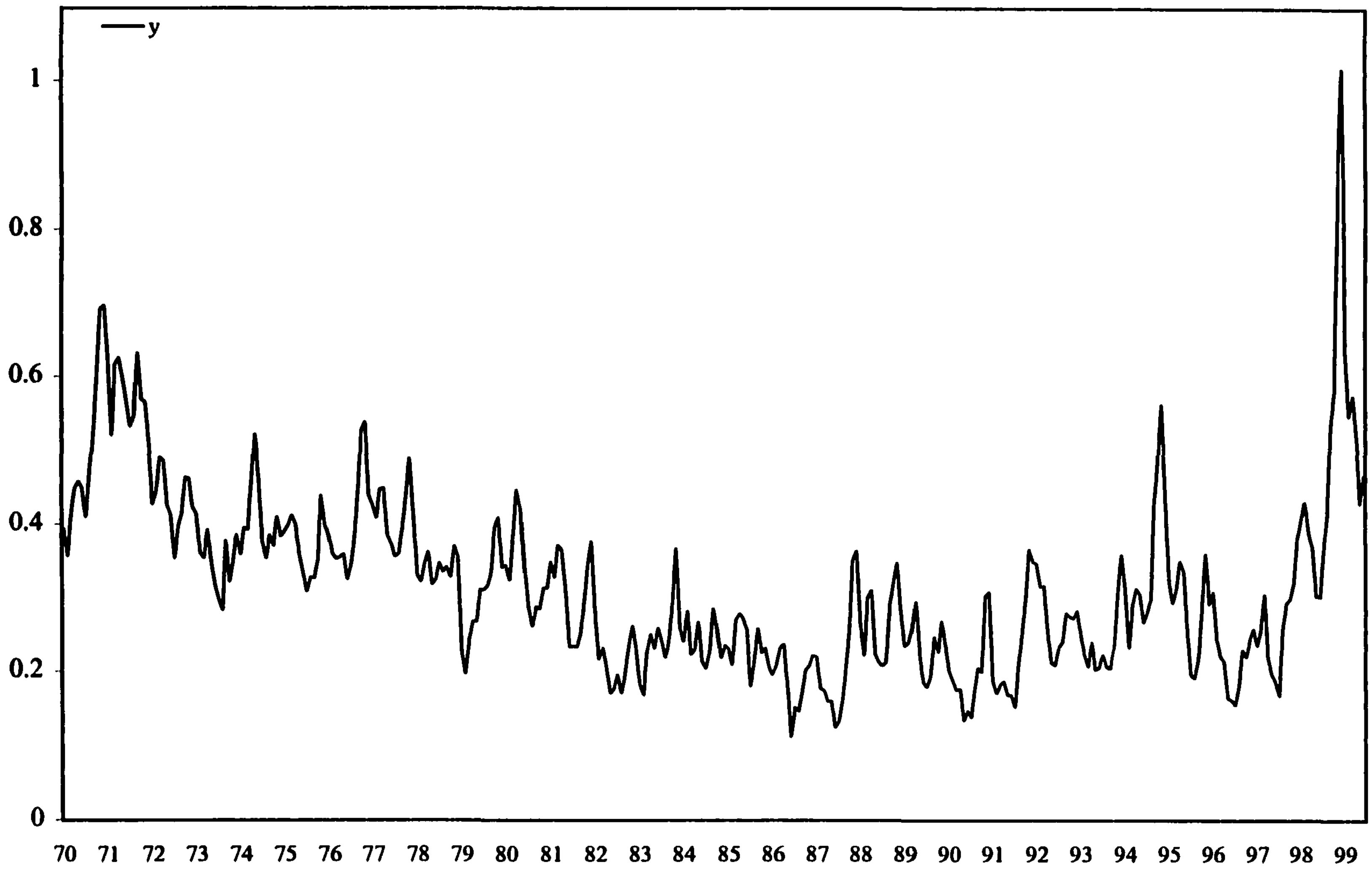


Figure 5.2 The Spread of Wholesale and Farm Prices of Pork in U.S., y (70:1 - 99:6)

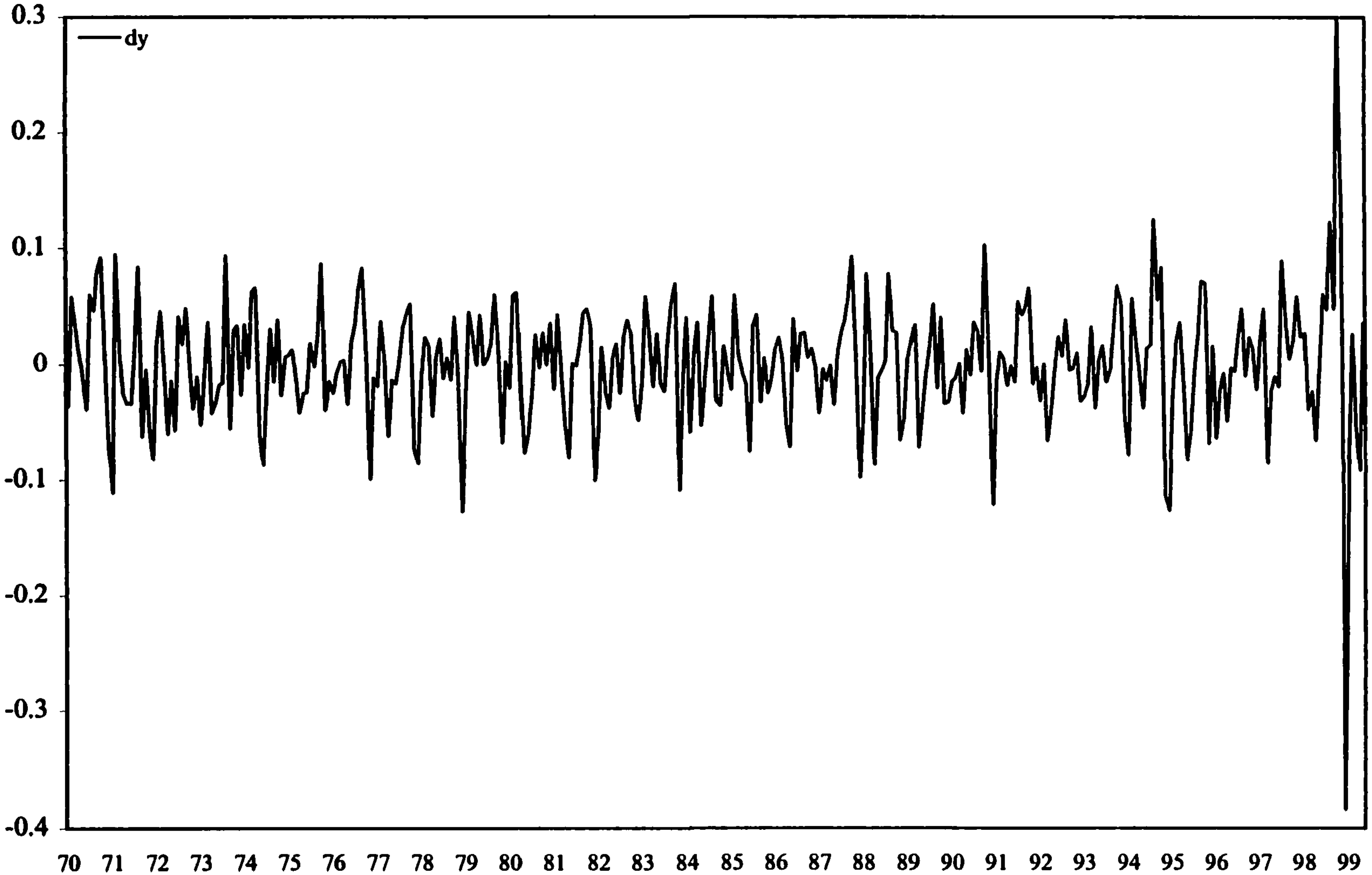


Figure 5.3 First-order difference of the Spread of Wholesale and Farm Prices of Pork in U.S., dy (70:1 - 99:6)

The unit root test results for y are summarized in Table 5.1. First, the Dickey-Fuller unit-root test statistics τ_μ for y, with value of -2.58, cannot reject the unit-root null hypothesis at the 5% significance level. The TAR unit-root test statistic, φ_μ, and F statistic, with values of 4.07 and 1.47, cannot reject unit-root and symmetric adjustment at the 5% significance level. Neither can the M-TAR unit-root test. Chan's (1993) method is used to get the consistent estimate of threshold. The TAR unit-root test statistic, φ_μ, and the F statistic, with the values of 6.40 and 6.05 respectively, are large enough to reject the null hypotheses of unit-root and symmetric adjustment at the 5% significance level, but the M-TAR unit-root test cannot.

However, if we disregard the period of 98:11 and 99:6, when there is a strike (the reason for doing so will be argued later), the Dickey-Fuller unit-root test statistics τ_μ, with value of -3.07, reject the unit-root null hypothesis at the 5% significance level. The TAR unit-root test statistic, φ_μ, with the value of 5.02, can reject a unit-root at the 5% level, but the value of the F statistic is not large enough to reject the null hypothesis of symmetric adjustment. Same for the M-TAR unit-root test. Chan's (1993) method is used to get the consistent estimate of threshold, and the same results can be got.

Since the results for the unit-root test of y are ambiguous, I estimate y with different kind of models and found that the summation of all coefficients is dangerously near 1 and there are ARCH effects (see Engle, 1982). If I estimate the spread for the period of 70:1-98:10, the summation of all coefficients is still dangerously near 1, but there are no ARCH effects. Thus, I consider the first-order difference of spread, dy.

Table 5.1 The Results for the Unit Root Tests of the Spread of the Pork Prices in U.S. [a]

Type of Test	Statistic	Period	
		70:1-99:6	70:1-98:10
Dickey-Fuller Unit-root	τ_μ	-2.58	-3.06
TAR Unit-root Test [c]	φ_μ	4.07	5.02
	$H_0: \rho_1 = \rho_2$	1.47	0.64
TAR Unit-root Test [c] with Consistent Threshold [e]	φ_μ	6.40	5.94
	$H_0: \rho_1 = \rho_2$	6.05	2.43
M-TAR Unit-root Test [c]	φ^*_μ	3.40	5.07
	$H_0: \rho_1 = \rho_2$	0.16	0.75
M-TAR Unit-root Test [c] with Consistent Threshold [e]	φ^*_μ	3.42	5.24
	$H_0: \rho_1 = \rho_2$	0.19	1.07

[a] All have 12 lags, selected by AIC. The SBC selected 2 lags, with significant serial correlation in the residuals.

[b] The numbers in the parentheses under the unit-root test statistics are the critical values at the 5% significance level.

[c] These are unit-root tests developed by Enders and Granger (1998).

[d] All numbers in the parentheses under the F-statistics for symmetry are the p-values.

[e] The consistent thresholds are estimated by Chan's (1993) method.

The Dickey-Fuller unit-root test statistics τ_μ for dy , with value of -5.59, reject the unit-root null hypothesis at the 1% significance level. The same conclusion can be reached for the period over 70:1-98:10.

When I estimate dy with different models, I found that all the models, except the TAR model with consistent estimate of the threshold, have ARCH effects. Then I tried to find a GARCH model, but I could not find an appropriate GARCH model. The reason for the ARCH effects is that the shock at the end of 1998 to the farm price of pork results in an abnormal large spread, dy and variance of spread during that period. So I decide to estimate dy for the period of 70:1-98:10 with a total of 346 observations.

5.3 In-Sample Estimation Results of dy

The results for the in-sample estimation are reported in Table 5.2. All the models selected are satisfying the conditions of significant parameters, no serial correlation of the residuals and no ARCH effects. The AR(12) model is the best model among all the ARMA models.

From the TAR model, the first lag of dy is not significant in each regime. I impose the coefficients of the second lag and the eleventh lag are the same in the two regime respectively because the F-tests for the null hypotheses of equality can not reject the null hypotheses. Even though the TAR model has a lower residual variance, the AIC and SBC are larger than those of the AR model. Chan's(1993) method is used to search a threshold of 0.06686 and estimate the consistent TAR model. This model has lower residual variance, AIC and SBC than the TAR model.

Table 5.2 Estimated Models for dy, the First-order Difference of the Spread of the Pork Prices in U.S. (71:4-98:10)

MODEL	ESTIMATED MODEL	AIC	SBC	RATIO[a]
AR(12)	$dy_t = 0.103\, dy_{t-1} - 0.193\, dy_{t-2} - 0.186\, dy_{t-4}$ (1.99)[b] (-3.77) (-3.70) $- 0.117\, dy_{t-9} + 0.194 dy_{t-11} + 0.206\, dy_{t-12}$ (-2.35) (3.82) (3.99)	-237.35	-214.54	
TAR	$dy_t = - 0.186\, dy_{t-2} + 0.242 dy_{t-11}$ (-3.52) (4.82) $+ 0.304 dy_{t-12} + 0.191 dy_{t-13},$ if $dy_{t-1} \geq 0$ (4.22) (2.77) $dy_t = - 0.186\, dy_{t-2} - 0.215\, dy_{t-3} - 0.226\, dy_{t-4}$ (-3.52) (-3.15) (-3.29) $- 0.197\, dy_{t-10} + 0.242 dy_{t-11}$, if $dy_{t-1} < 0$ (-2.76) (4.83)	-236.34	-209.72	0.987
TAR-C[c]	$dy_t = 0.367\, dy_{t-1} - 0.485\, dy_{t-5} + 1.018\, dy_{t-8}$ (2.44) (-2.48) (3.87) $- 1.090 dy_{t-9} - 1.518\, dy_{t-13},$ (-5.05) (-4.25) if $dy_{t-1} \geq 0.06686$ $dy_t = 0.134\, dy_{t-1} - 0.201 dy_{t-2} - 0.157 dy_{t-4}$ (2.32) (-3.86) (-3.16) $+ 0.256\, dy_{t-11} + 0.209 dy_{t-12}$ (5.02) (4.07) $+ 0.117\, dy_{t-13}$, if $dy_{t-1} < 0.06686$ (2.22)	-255.91	-210.29	0.902

a Ratio is the ratio of the residual variances of the nonlinear model and of the linear AR model.

[b] All numbers in parentheses are T-statistics.

[c] TAR-C and M-TAR-C stands for the TAR and M-TAR models using a consistent estimate of the threshold (Chan, 1993).

Table 5.2 (continued)

MODEL	ESTIMATED MODEL[a]	AIC	SBC	RATIO[b]
M-TAR	$dy_t = -0.145\, dy_{t-4} - 0.154\, dy_{t-9} + 0.136\, dy_{t-10}$ (-2.17) (-2.37) (2.28) $+ 0.284\, dy_{t-12}$, if $d^2y_{t-1} \geq 0$ (3.96) $dy_t = - 0.270 dy_{t-2} - 0.210 dy_{t-3} - 0.251 dy_{t-4}$ (-3.42) (-2.65) (-3.42) $-0.263\, dy_{t-6} - 0.161\, dy_{t-9} - 0.185\, dy_{t-10}$ (-3.28) (-2.07) (-2.19) $+ 0.279\, dy_{t-11} + 0.146 dy_{t-12}$, if $d^2y_{t-1} < 0$ (3.64) (2.02)	-242.93	-197.31	0.931
M-TAR-C[d]	$dy_t = 0.006 - 0.191\, dy_{t-4} - 0.159\, dy_{t-9}$ (2.11) (-3.95) (-3.27) $+ 0.142\, dy_{t-10} + 0.201\, dy_{t-12}$, (2.40) (4.01) if $d^2y_{t-1} \geq 0.0028$ $dy_t = - 0.254 dy_{t-2} - 0.211 dy_{t-3} - 0.191 dy_{t-4}$ (-3.35) (-2.83) (-3.95) $-0.243\, dy_{t-6} - 0.159\, dy_{t-9} - 0.172\, dy_{t-10}$ (-3.26) (-3.27) (-2.18) $+ 0.283\, dy_{t-11} + 0.201 dy_{t-12}$, (3.93) (4.01) if $d^2y_{t-1} < 0.0028$	-251.73	-209.90	0.920

Table 5.2 (continued)

MODEL	ESTIMATED MODEL[a]	AIC	SBC	RATIO[b]
GAR	$dy_t = -0.287\, dy_{t-2} + 0.239 dy_{t-11} + 0.473\, dy_{t-12}$ (-5.80) (4.27) (7.38) $+ 15.848 dy_{t-1}^3 + 3.804 dy_{t-6}\, dy_{t-10}$ (2.44) (4.11) $- 36.615\, dy_{t-2}^2 dy_{t-5} - 63.235\, dy_{t-9}^2 dy_{t-12}$ (-2.84) (-4.17) $+ 37.644 dy_{t-1}^2 dy_{t-8} - 59.533\, dy_{t-2}^2 dy_{t-9}$ (2.78) (-4.21) $-68.332\, dy_{t-3}^2 dy_{t-12} - 28.670\, dy_{t-7} dy_{t-8}^2$ (-3.77) (-2.59) $-37.185\, dy_{t-11} dy_{t-13}^2 - 74.113\, dy_{t-3} dy_{t-9}^2$ (-2.65) (-4.52) $-32.357\, dy_{t-4} dy_{t-11}^2 + 42.477\, dy_{t-1} dy_{t-9}^2$ (-2.25) (2.51) $-43.427\, dy_{t-4} dy_{t-13}^2 - 54.599\, dy_{t-1} dy_{t-12}^2$ (-3.40) (-4.04)	-308.43	-243.79	0.743
EAR	$dy_t = 0.103\, dy_{t-1} - 0.194 \exp(-dy_{t-1}^2) dy_{t-2}$ (1.99) (-3.77) $- 0.186\, dy_{t-4} - 0.117\, dy_{t-9}$ (-3.70) (-2.35) $+ 0.194\, dy_{t-11} + 0.206 dy_{t-12}$ (3.82) (3.99)	-237.37	-214.56	0.990
BILINEAR	$dy_t = 0.093\, dy_{t-1} - 0.184\, dy_{t-2} - 0.187\, dy_{t-4}$ (2.01) (-4.13) (-4.11) $- 0.179\, dy_{t-9} + 0.203 dy_{t-11} + 0.181\, dy_{t-12}$ (-4.21) (4.39) (3.85) $+ 9.812 dy_{t-10}\, \hat{e}_{t-9} - 5.732\, dy_{t-9}\, \hat{e}_{t-10}$ (5.53) (-3.60) $+ 2.468\, dy_{t-10}\, \hat{e}_{t-6}$ (1.80)	-262.22	-224.20	0.857

For the M-TAR model, even though it has lower residual variance and AIC, the SBC is larger than that of the AR model. Chan's(1993) method is used to search a threshold of 0.0028 and estimate the consistent M-TAR model. This model has lower residual variance, AIC and SBC than the M-TAR model.

The GAR model adds many third-order crossproducts to it. This model has the smallest residual variance, AIC and SBC among all the models. The explanation of it is not easy.

By adding an exponential coefficient to the second lag, the EAR model is slightly better than the AR model.

The bilinear model is much better than the AR model by adding crossproducts of the lags of the dy and the residuals, which means that the change of the spread depends on its past observations and the correlation of its past observations and the shocks.

5.4 Out-of-Sample Forecasting Performance Results of dy

The 1-step ahead forecast of the first-order difference of the spread of the pork farm and wholesale prices is applied over the period of 1987:1-1998:10 with a total of 142 forecast values. The forecast of each model is estimated recursively. The methodology and the test statistics are the same as those of the term structure of interest rates.

The results of the out-of-sample forecasting are reported in Table 5.3. The forecasts of the M-TAR and GAR models are significantly biased downward. There is no significant forecast bias for other models. From the MSPE ratios, the GAR model generates the lowest MSPE among all the models. The MSPE of bilinear model is substantial lower than that of

Table 5.3 Bias and MSPE Ratios of Different Models for the First-order Difference of the Spread of the Pork Prices in U.S. (87:1-98:10)

MODEL	BIAS [a]	MSPE RATIOS [b]
AR(12)	-0.003 (-0.81)	
TAR	-0.002 (-0.71)	1.004 (0.99)
TAR-C[c]	-0.003 (-0.88)	1.098 (0.95)
M-TAR	-0.008 (-2.47)	1.010 (0.99)
M-TAR-C[c]	-0.005 (-1.61)	1.014 (0.98)
GAR	-0.006 (-2.09)	0.780 (0.83)
EAR	-0.003 (-0.81)	0.999 (0.98)
BILINEAR	-0.003 (-0.96)	0.893 (0.83)

[a] Bias is calculated by regressing the forecast error on a constant; the number below is its t-statistic.

[b] MSPE Ratios are the ratios of MSPEs of the nonlinear models to that of the AR model; the numbers in parentheses are the p-values for the Mizrach's (1995) robust forecast comparison statistic, testing that the MSPE ratio equals 1.

[c] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan,1993).

the AR model. The MSPE of the EAR model is slightly lower than that of the AR model. However, from the p-values of the Mizrach's (1995) robust forecast comparison statistic, since all of them are relatively large, the reductions in MSPE are not statistically significant. All other models have larger MSPEs than that of the AR model, although there are no statistical difference between them. One phenomenon needs to be pointed out is the absolute consistency of in-sample estimation and out-of-sample forecast. That is, those models that are better for the in-sample estimation than the AR model are also better for the out-of-sample forecast, and verse versa. Comparing the results here with the simulation results of AR(2) models, it is not very likely that the dominance of the nonlinear models over the AR model for the spread of pok prices results from overfitting.

5.5 Conclusions

The results are summarized in Table 5.4. The GAR model is the best model for the first-order difference of the spread of the farm and wholesale pork prices for both the in-sample estimation and out-of-sample forecast. The bilinear model is substantially better than the AR model for both the in-sample estimation and out-of-sample forecast. The EAR model is slightly better than the AR model for both the in-sample estimation and out-of-sample forecast However, the dominance of the GAR, EAR and bilinear models for the out-of-sample forecast over the AR model is not statistically significant, which may be explained by the previous finding that the extent of price asymmetry is generally modest.

Table 5.4 The Summary of the Estimation and Forecast Results for the First-order Difference of the Spread of the Pork Prices in U.S.

Models	First-order Difference of the Spread	
	In-sample	Out-of-sample
TAR	W [a]	W
TAR-C [b]	A[c]	W
M-TAR	A	W
M-TAR-C [b]	A	W
EAR	B[d]	B
GAR	**B** [e]	**B**
Bilinear	B	B

[a] W means that the nonlinear model is worse than the ARMA model in term of the AIC and SBC for in-sample estimation or the MSPE for out-of-sample forecast.

[b] TAR-C and M-TAR-C stand for the TAR and M-TAR models using a consistent estimate of the threshold (Chan,1993).

[c] A stands for ambiguity, i.e., the nonlinear model has a smaller AIC, but a larger SBC than the ARMA model.

[d] B means that the nonlinear model is better than the ARMA model in term of the AIC and SBC for in-sample estimation or the MSPE for out-of-sample forecast.

[e] **B** means the best model among all the models.

CHAPTER 6 CONCLUSIONS AND FUTURE WORK

6.1 Main Conclusions

For AR processes, if the AIC and SBC criteria are used to select models, the possibility of overfitting is very high since the non-linear models and other linear models are very likely to have lower AIC and SBC. However, the MSPE for one-step ahead out-of-sample forecast can be used to identify the true AR processes. For TAR processes, the AIC can be used to identify the TAR-C models. The SBC and MSPE can identify the TAR process only if the difference of the persistence between the two regimes is large enough. Underfitting and misspecification are very likely to happen for a TAR process with small difference of the persistence between the two regimes.

However, if we don't know the true AR or TAR process, the MSPE can't select the AR or TAR models in most cases. Thus, none of the AIC, SBC and MSPE can select the AR model for a given AR process with unknown order. For the TAR process, the AIC can consistently identify the TAR-C process and the SBC can identify the TAR-C process only if the difference of the persistency is large enough.

For both the term structure of interest rates and the spread of wholesale and retail pork prices in U.S., there are non-linear time series models can do better than the conventional ARMA models for both in-sample estimation and out-of-sample forecast. Also, it is very unlikely that the dominance of the nonlinear time series models results from overfitting for both the term structure of interest rates and the spread of wholesale and retail pork prices in U.S.. Thus, non-linear time series models are very useful for estimating and forecasting the non-linear time series.

6.2 Future Work

Many issues can be investigated further about the study of non-linear time series models. The followings are some examples.

A natural extension to those univariate studies is to investigate the non-linearity in a multivariate context. Towards that end, Enders and Siklos (1999) developed a generalization of Engle-Granger (1987) testing procedure that allowing the TAR or M-TAR adjustment toward a cointegrating vector and applied an error-correction model with TAR and M-TAR adjustments to the short-term and long-term interest rate. Anderson (1994) estimated a smooth transition error-correction model of the U.S. treasury bill market. Granger and Lee (1989) investigated U.S. production, sale and inventory asymmetric relationship using multicointegration and asymmetric error-correction models. Given that many economic variables display asymmetric behavior, non-linear error-correction models will be a promising area to explore. Then, impulse response functions and variance decompositions can be used to study the adjustment processes to shocks.

The research may be extended to conduct multi-step ahead forecast and check how non-linear time series models behave for different multi-step ahead forecasts.

The Monte Carlo study of overfitting and forecasting may also be extended to multivariate cases.

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